Bulletin of the Seismological Society of America, Vol. 85, No. 2, pp. 661–664, April 1995

Reply to Comments on "Crustal Anisotropy from Local Observations of Shear-Wave Splitting in West Bohemia, Czech Republic" by G. H. R. Bokelmann and J. Kawahara: Can the Hudson Crack Model Describe Behavior of Real Cracks?

by Václav Vavryčuk[1]

Bokelmann (1995) disagrees with my conclusion that Hudson's model of dry or water-filled parallel cracks does not fit shear-wave splitting data observed in the West Bohemia region and that the real crack behavior is not adequately described by these models (Vavryčuk, 1993). He argues that the orientation of the symmetry axis (horizontal axis with azimuth 31°) may not be appropriate, since it is derived from the fast shear-wave polarizations without dense ray coverage. He states that in particular the dip of the symmetry axis is not well constrained and proposes an alternative model—the Hudson crack model with an inclined symmetry axis, claiming that this model is in coincidence with the observed data. In this article, I shall demonstrate that neither the alternative model proposed by Bokelmann nor other models of dry or water-filled cracks with an inclined symmetry axis fit the West-Bohemian shear-wave splitting data. Finally, I will discuss a modification of Hudson's model suggested by Kawahara (1995), who proposes not to interpret an overall anisotropy of cracked media by standard crack parameters (aspect ratio of ellipsoidal cracks, elastic parameters of an infilling material) but to consider the response U_{ij} of a crack to the stress field as a phenomenological quantity. Only after this modification can a fit with the shear-wave splitting data be obtained. I will present optimum values of U_{11} and U_{33} for such a model involving the Hudson second-order perturbation theory.

Bokelmann (1995) is right that the axis direction is not determined precisely. The uncertainty arises from inaccuracy in the measurement of polarization directions of split S waves and because the focal sphere is not sufficiently covered with rays. Moreover, the fast shear-wave polarization for rays inside the shear-wave window is not much sensitive to a dip of the symmetry axis, particularly for values in the range of 0° to 30° (e.g., Crampin and Booth, 1985). Therefore, the actual symmetry axis can really differ from the proposed one, deviating somewhat from the horizontal direction. For these reasons, other directions of axis should also be taken into account, but for simplicity, only the strictly horizontal case is treated in Vavryčuk (1993).

[1]Present address: Hiroshima University, Department of Earth and Planetary Systems Science, Higashi-Hiroshima 724, Japan.

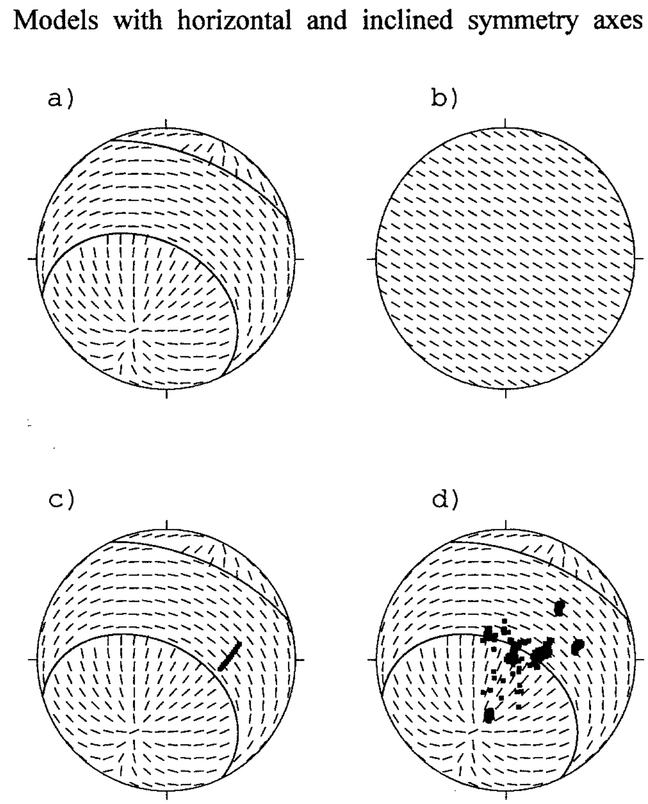

Figure 1. Comparison of proposed anisotropy models. Lower-hemisphere, equal-area projections of the S_1-wave polarization predicted by (a), (c), (d) model with an inclined symmetry axis (Bokelmann's model) and (b) model with a horizontal symmetry axis (Vavryčuk's model). The Bokelmann model is shown (c) with the data for which a directional variation of the delay time was observed (station VAC) and (d) with the data for which polarization directions of split S waves were measured (stations VAC, TIS, SEL, HOH, and KOS). The line defines rays with no shear-wave splitting ("line shear-wave singularity") and separates regions with faster SP or SR wave. In (c) we do not consider a steepening of rays beneath the station, thus approximating an averaged ray direction, since the delay time is integrated along a whole ray. In (d) we use dynamic incidence angles taking into account that the polarization direction is a local property reflecting anisotropy rather beneath the station. The effects of a free surface on the S-wave polarization are not considered.

Let me now discuss crack models with an inclined symmetry axis and examine if they can fit the shear-wave splitting data. Vavryčuk (1993) shows that under an assumption of a horizontal symmetry axis *no shear-wave splitting is observed for rays approximately perpendicular to the axis*. This is in strong disagreement with the Hudson model of dry or water-filled parallel cracks (Hudson, 1980, 1981). The model of dry cracks predicts no splitting only for rays parallel to the symmetry axis and the model of saturated water-filled cracks also for rays inclining 60° from the axis. In contrast, both models predict maximum delay times between S_1 and S_2 for rays 90° from the axis (see Crampin, 1984). A fit cannot be obtained for the *dry cracks* even with an inclined symmetry axis because the axis would have to be parallel to the rays with no splitting. These rays come very steeply to a station, implying that the axis should deviate 60° to 70° from the horizontal. The fast shear-wave polarization would have to be strongly dependent on a ray direction in such a case, thus resulting in S-wave polarizations quite contradictory to our observations. For *saturated water-filled cracks*, a fit can be obtained by inclining the symmetry axis, as proposed by Bokelmann, by 40° to 50° from the horizontal (strike of the symmetry plane 114°, dip 50°, crack density 0.05, background P-wave velocity 5.8 km/sec, velocity ratio 1.73). As a consequence of such a large deviation of the symmetry axis from the horizontal direction, the S_1-wave polarization would depend on the ray direction (see Fig. 1a, 1b). As regards the VAC station, we cannot see such a prominent dependence, and a satisfactory fit of polarizations both measured and predicted by Bokelmann as well as of the delay time variations is achieved (Fig. 1c): the rays from the south almost touch the line singularity, and the polarizations essentially are consistent with the observations. The model predicts, however, the *SP* wave to be the faster wave for rays deviating from the axis up to 60°. Figure 1d shows the data used in Vavryčuk (1993) together with additional 1991 to 1993 S-wave polarization data from the local station KOS. From this figure, a reverse order of split S waves should be observed for a large number of events, but we detected no such cases in the entire data set of more than 500 events. Only the S_1 wave could be measured for many events, and always it was the *SR* wave (relative to the axis with azimuth 31°). This fact is also demonstrated by Figure 2, which shows examples of splitting for rays with different azimuths and incidence angles. This figure brings clear evidence that the S_1 wave is always the *SR* wave and that the directions of polarization do not depend on the ray!

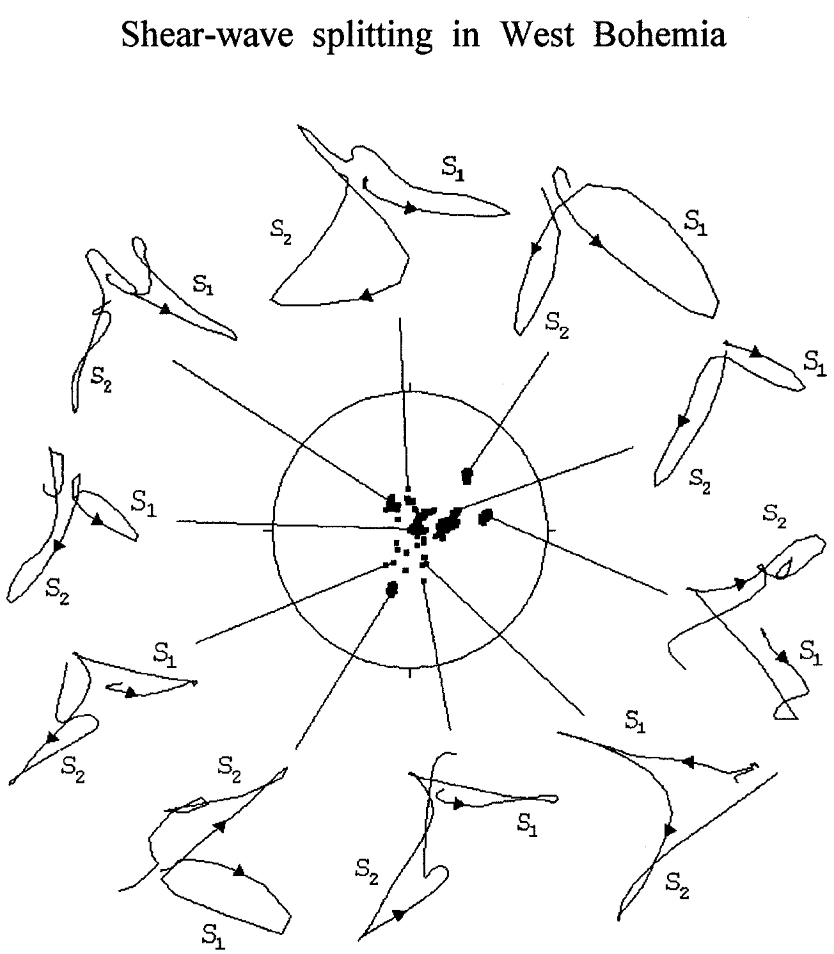

Figure 2. Examples of shear-wave splitting observed in the West Bohemia region. S-wave polarizations are shown for events (clockwise, from the *top*): 23 January 1986 at 07:09:16 (HOH, M_D = 1.6); 20 January 1986 at 23:43:21 (VAC, M_L = 1.2); 30 December 1985 at 21:49:56 (SEL, M_L = 2.6); 5 September 1991 at 12:28:56 (KOS, M_L = 1.3); 6 September 1991 at 23:17:55 (KOS, M_L = 2.4); 6 February 1986 at 09:17:15 (TIS, M_L = 2.6); 30 August 1991 at 02:33:26 (KOS, M_L = 1.5); 23 December 1989 at 22:07:53 (KOS, M_L = 0.4); 23 September 1991 at 19:21:51 (KOS, M_L = 0.6); 4 May 1986 at 15:41:55 (KOS, M_L = 1.3).

It must be said, however, that Bokelmann cannot be blamed for neglecting this fact because this evidence is missing in Vavryčuk (1993).

Similar to parallel cracks, the shear-wave splitting data cannot be explained even by the model of dry or water-filled cracks with a random distribution of normals about the axis direction (Peacock and Hudson, 1990), by cracks with coplanar normals (Peacock and Hudson, 1990), or by cracks with nonzero aspect ratios (Douma, 1988; Douma and Crampin, 1990). As was shown in Vavryčuk (1993), the successful fracture model exhibits almost no response of fractures to shear stress. It implies the existence of remarkably high friction or another shear interaction between both sides of cracks. This interaction can be interpreted, for example, by irregular shapes of cracks as is suggested by Vavryčuk (1993). In all the crack models mentioned above, however, *no friction and smooth-faced ellipsoidal cracks are assumed* just because of mathematical reasons but without any experimental justification. Note that the "no friction" condition ("stress-free" crack) is not applied only by Hudson but is used quite standardly, even in earlier theoretical articles on cracks (e.g., Garbin and Knopoff, 1973, 1975a, 1975b; Anderson *et al.*, 1974; O'Connell and Budiansky, 1974). Obviously, such a restrictive assumption affects overall properties of the model considerably and can lead to discrepancies with data.

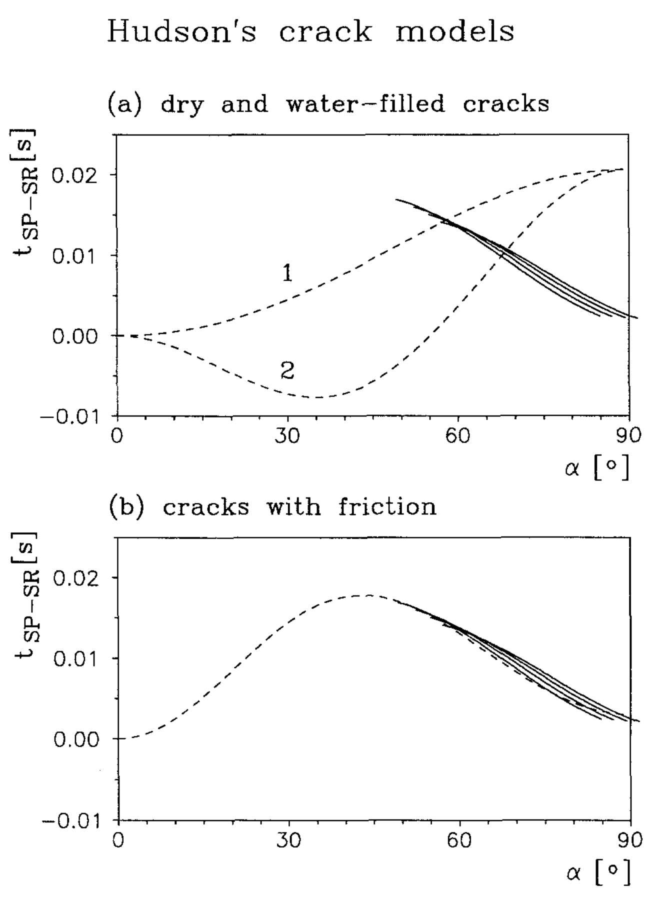

Figure 3. Comparison of observed delay time data (solid line) with Hudson's crack models (dashed line). (a) Model 1: dry cracks ($U_{11} = 2.0$, $U_{33} = 2.3$); model 2: water-filled cracks ($U_{11} = 0$, $U_{33} = 2.3$). (b) Model of cracks with friction ($U_{11} = 2.0$, $U_{33} = 0.34$). Elasticity tensor for model (b) exactly coincides with that for the Schoenberg–Douma fracture model for values $E_N = 0.40$ and $E_T = 0.02$. In all crack models, vertical parallel cracks are assumed with azimuth of normal 31°, crack density 0.058, background velocity 5.8 km/sec, and velocity ratio 1.73. The delay time is normalized to 1 km of the propagation distance. Note that an incorrect notation t_{SR-SP} instead of t_{SP-SR} is used in Vavryčuk (1993).

Peacock and Hudson (1990) pointed out an enormous mathematical simplification done in crack models, realizing that the real cracks are far from being smooth-faced ellipsoids and presumably have a great variety of sizes, shapes, and orientations. Nevertheless, they advocate the concept of simplified cracks, since crack shapes more complicated than oblate spheroids are analytically intractable and reconciling of the Hudson model any further with real cracks would lead to sacrificing its simplicity. This difficulty seems to be overcome by Kawahara (1995), who proposes the quantities U_{11} and U_{33} (Hudson, 1988, 1991) as *independent* and *pure phenomenological*, reflecting a real crack behavior in the stress field. The quantities U_{11} and U_{33} are no longer connected to a response of mathematically well defined but geologically absurd empty or saturated ellipsoidal cracks. In this aspect, his approach is similar to that in fracture models (Schoenberg and Douma, 1988), where fracture parameters are not assumed to be calculated theoretically, but evaluated by an experiment. Adopting this idea and assuming a weak response of a crack to the shear stress by a small value of U_{33} (crack normal is in the x_1 direction), a good fit with the data is indeed obtained (see Fig. 3). Applying Hudson's second-order perturbation formulas, we get an optimum crack model with values $eU_{11} = 11.6 \cdot 10^{-2}$ and $eU_{33} = 2.0 \cdot 10^{-2}$. It implies that in crack models *an actual crack behavior should be investigated in experimental studies* but not assumed *a priori* by applying such mathematical notions as the "dry" (stress-free) or "water-filled" (shear-stress-free) cracks. Although so far these models have been considered as relevant models describing overall properties of real cracked media properly, they should be viewed as *theoretical examples* whose practical use is questionable.

Acknowledgments

The author thanks A. Boušková and J. Horálek from Geophysical Institute of Czech Academy of Sciences for their help in processing of additional data from the KOS station, J. Kawahara for discussions on the subject, and A. Plešinger and K. Yomogida for critically reading the manuscript and for their comments.

References

Anderson, D. L., B. Minster, and D. Cole (1974). The effect of oriented cracks on seismic velocities, *J. Geophys. Res.* **79**, 4011–4015.

Bokelmann, G. H. R. (1995). Comment on "Crustal anisotropy from local observations of shear-wave splitting in West Bohemia, Czech Republic," by V. Vavryčuk, *Bull. Seism. Soc. Am.* **85**, no. 2, 653–657.

Crampin, S. (1984). Effective anisotropic elastic constants for wave propagation through cracked solids, *Geophys. J. R. Astr. Soc.* **76**, 135–145.

Crampin, S. and D. C. Booth (1985). Shear-wave polarizations near the North Anatolian fault, II, Interpretation in terms of crack-induced anisotropy, *Geophys. J. R. Astr. Soc.* **83**, 75–92.

Douma, J. (1988). The effect of the aspect ratio on crack-induced anisotropy, *Geophys. Prospect.* **36**, 614–632.

Douma, J. and S. Crampin (1990). The effect of a changing aspect ratio of aligned cracks on shear wave vertical seismic profiles: a theoretical study, *J. Geophys. Res.* **95**, no. B7, 11.293–11.300.

Garbin, H. D. and L. Knopoff (1973). The compressional modulus of material permeated by a random distribution of circular cracks, *Q. Appl. Math.* **30**, 453–464.

Garbin, H. D. and L. Knopoff (1975a). The shear modulus of material permeated by a random distribution of free circular cracks, *Q. Appl. Math.* **33**, 296–300.

Garbin, H. D. and L. Knopoff (1975b). Elastic moduli of a medium with liquid-filled cracks, *Q. Appl. Math.* **33**, 301–303.

Hudson, J. A. (1980). Overall properties of a cracked solid, *Math. Proc. Cambridge Philos. Soc.* **88**, 371–384.

Hudson, J. A. (1981). Wave speeds and attenuation of elastic waves in material containing cracks, *Geophys. J. R. Astr. Soc.* **64**, 133–150.

Hudson, J. A. (1988). Seismic wave propagation through material containing partially saturated cracks, *Geophys. J.* **92**, 33–37.

Hudson, J. A. (1991). Overall properties of heterogeneous material, *Geophys. J. Int.* **107**, 505–511.

Kawahara, J. (1995). Comment on "Crustal anisotropy from local observations of shear-wave splitting in West Bohemia, Czech Republic," by V. Vavryčuk, *Bull. Seism. Soc. Am.*, **85**, no. 2, 658–660.

O'Connell, R. J. and B. Budiansky (1974). Seismic velocities in dry and saturated cracked solids, *J. Geophys. Res.* **79**, 5412–5426.

Peacock, S. and J. A. Hudson (1990). Seismic properties of rocks with distributions of small cracks, *Geophys. J. Int.* **102**, 471–484.

Schoenberg, M. and J. Douma (1988). Elastic wave propagation in media with parallel fractures and aligned cracks, *Geophys. Prospect.* **36**, 571–590.

Vavryčuk, V. (1993). Crustal anisotropy from local observations of shear-wave splitting in West Bohemia, Czech Republic, *Bull. Seism. Soc. Am.* **83**, 1420–1441.

Geophysical Institute
Czech Academy of Sciences
Boční II/1401, Praha 4, 141 31, Czech Republic

Manuscript received 7 February 1994.