Geophys. J. Int. (1999) **138,** 553–562

Weak-contrast reflection/transmission coefficients in weakly anisotropic elastic media: *P*-wave incidence

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Accepted 1999 April 1. Received 1998 December 14; in original form 1998 March 16

SUMMARY

We present approximate displacement and energy *PP* and *PS* reflection/transmission coefficients for weak-contrast interfaces in general weakly anisotropic elastic media. The coefficients were obtained by applying first-order perturbation theory and then expressed in a compact and relatively simple form. The formulae can be used for arbitrary orientations of the incidence plane and interface, without the need to transform the elasticity parameters to a local Cartesian coordinate system. The accuracy of the approximate formulae is illustrated for the *PS* reflection coefficient for two synthetic models. For these models, we also study the possibility of using the approximate *PP* reflection coefficient in the inverse problem.

Key words: elastic wave theory, scattering, seismic anisotropy, seismic waves.

1 INTRODUCTION

The calculation of the reflection/transmission (R/T) coefficients of plane waves at a plane interface is more complex in anisotropic media than in isotropic media (see Keith & Crampin 1977). Under isotropy, the problem can be separated into the independent reflection/transmission of *P–SV* waves and *SH* waves. Under anisotropy, the incident wave generates six waves: *P*, *S*1 and *S*2 reflected waves, and *P*, *S*1 and *S*2 transmitted waves, and thus the problem is represented, in general, by six non-separable linear algebraic equations in six unknown R/T coefficients. An explicit analytic solution of such a system is cumbersome, even if a simple anisotropy such as transverse isotropy is considered (see Daley & Hron 1977). Therefore, a numerical solution of the system of equations is commonly used (see Fryer & Frazer 1984; Gajewski & Pšenčík 1987). An alternative method is to use approximate analytical formulae derived for a weak-contrast interface in weakly anisotropic media. Since in many practical applications the anisotropy and contrast across the interface are not strong (Thomsen 1986), such formulae are often sufficiently accurate. Moreover, the formulae are relatively simple and comprehensible. The first attempt to determine linearized R/T coefficients for weak anisotropic media was made by Thomsen (1993), who derived formulae for transversely isotropic half-spaces with axes of symmetry perpendicular to the interface. Rueger (1997) corrected Thomsen's results and found linearized R/T coefficients for transversely isotropic half-spaces with axes of symmetry parallel to the interface, but only in the symmetry plane. Haugen & Ursin (1996) derived *PP* reflection coefficients for a further special orientation of the symmetry axes of transversely isotropic half-spaces. Zillmer *et al.* (1998) derived *PP* reflection coefficients for a horizontal interface in general weakly anisotropic media, and *SVSV* and *SHSH* reflection coefficients for waves propagating in a symmetry plane. Pšenčík & Vavryčuk (1998) and Vavryčuk & Pšenčík (1998) derived *PP* displacement R/T coefficients for a horizontal interface in general weakly anisotropic media. In this paper, we extend the above results by deriving formulae for *PP* as well as *PS* R/T coefficients for arbitrarily oriented weak-contrast interfaces in general weakly anisotropic elastic media. For two models, we study the accuracy of the derived formulae and the possibility of applying the formulae in the inverse problem.

2 REFLECTION/TRANSMISSION OF PLANE WAVES IN ANISOTROPIC MEDIA

Let us consider two homogeneous anisotropic half-spaces separated by a plane interface Σ specified by normal $\mathbf{\nu}$. We assume that the half-spaces are in welded contact at interface Σ. The half-space, into which normal $\mathbf{\nu}$ points, will be referred to as half-space (1); the other half-space will be referred to as half-space (2). The half-spaces are characterized by densities $\rho^{(I)}$, $I=1, 2$, and by the density-normalized elasticity tensors $a^{(I)}_{ijkl}$, $I=1, 2$. The incident harmonic plane wave propagates in half-space (1) and generates six plane harmonic waves, namely reflected *P*, *S*1 and *S*2, and transmitted *P*, *S*1 and *S*2. The displacement and traction vectors $\mathbf{u}(\mathbf{x}, t)$ and $\mathbf{T}(\mathbf{x}, t)$ of any of the waves can be expressed as follows:

$$\mathbf{u}^{(N)}(\mathbf{x}, t)=c^{(N)}\mathbf{g}^{(N)}\exp[i\omega(t-\mathbf{p}^{(N)}\cdot\mathbf{x})]\,,$$
$$\mathbf{T}^{(N)}(\mathbf{x}, t)=c^{(N)}\boldsymbol{\sigma}^{(N)}\exp[i\omega(t-\mathbf{p}^{(N)}\cdot\mathbf{x})]\,, \tag{1}$$

where $c^{(N)}$ is the scalar amplitude, $\mathbf{g}^{(N)}$ is the unit polarization vector, $\mathbf{p}^{(N)}$ is the slowness vector, ω is the circular frequency, and t is time. The superscript N denotes the type of wave: $N=0$ corresponds to the incident wave; $N=1$, 2 and 3 to the reflected P, $S1$ and $S2$; and $N=4$, 5 and 6 to the transmitted P, $S1$ and $S2$, respectively. If the scalar amplitude of the incident wave, $c^{(0)}$, is equal to 1, the amplitude $c^{(N)}$ of the Nth scattered wave represents its displacement R/T coefficient. $\boldsymbol{\sigma}^{(N)}$ is the amplitude-normalized traction vector of the Nth wave:

$$\sigma_i^{(N)}=\rho^{(I)}a_{ijkl}^{(I)}v_j g_k^{(N)} p_l^{(N)}. \tag{2}$$

The superscript $I=1, 2$ identifies the half-space in which the wave propagates. The displacement and traction at the interface must satisfy the boundary conditions requiring their continuity across the interface. If we introduce a 6-vector $\mathbf{d}^{(N)}$ for each wave,

$$\mathbf{d}^{(N)}=\pm\begin{bmatrix}\mathbf{g}^{(N)}\\ \boldsymbol{\psi}^{(N)}\end{bmatrix}, \tag{3}$$

where the plus sign stands for the reflected and the minus sign for the transmitted and incident waves, we can express the boundary conditions by the following equation:

$$\mathbf{Dc}=\mathbf{d}^{(0)}. \tag{4}$$

Subsequently, we obtain

$$\mathbf{c}=\mathbf{D}^{-1}\mathbf{d}^{(0)}. \tag{5}$$

$\mathbf{D}$ is the 6×6 matrix called the displacement–stress matrix, $\mathbf{d}^{(N)}$ is the displacement–stress vector of the Nth wave, and $\mathbf{c}$ is the 6-vector of the displacement R/T coefficients:

$$\begin{aligned}\mathbf{D}&=\begin{bmatrix}\mathbf{g}^{(1)} & \mathbf{g}^{(2)} & \mathbf{g}^{(3)} & -\mathbf{g}^{(4)} & -\mathbf{g}^{(5)} & -\mathbf{g}^{(6)}\\ \boldsymbol{\sigma}^{(1)} & \boldsymbol{\sigma}^{(2)} & \boldsymbol{\sigma}^{(3)} & -\boldsymbol{\sigma}^{(4)} & -\boldsymbol{\sigma}^{(5)} & -\boldsymbol{\sigma}^{(6)}\end{bmatrix}\\ &=[\mathbf{d}^{(1)}\quad \mathbf{d}^{(2)}\quad \mathbf{d}^{(3)}\quad \mathbf{d}^{(4)}\quad \mathbf{d}^{(5)}\quad \mathbf{d}^{(6)}],\\ \mathbf{c}&=(c^{(1)}\quad c^{(2)}\quad c^{(3)}\quad c^{(4)}\quad c^{(5)}\quad c^{(6)}]^{\mathrm{T}}\\ &=[R^P\quad R^{S1}\quad R^{S2}\quad T^P\quad T^{S1}\quad T^{S2}]^{\mathrm{T}}.\end{aligned}$$

Instead of the *displacement R/T coefficients*, coefficients normalized to the energy flux normal to Σ are sometimes used. We call these the *energy R/T coefficients* and define them as the ratio of the normal component of the energy flux of the Nth scattered wave to that of the incident wave:

$$c_{\mathrm{E}}^{(N)}=\left|\frac{\mathbf{w}^{(N)}\cdot\mathbf{v}}{\mathbf{w}^{(0)}\cdot\mathbf{v}}\right|,$$

where $\mathbf{w}^{(0)}$ and $\mathbf{w}^{(N)}$ are the vectors of the energy flux of the incident and Nth scattered waves, and $\mathbf{v}$ is the normal to the interface. Since

$$c_{\mathrm{E}}^{(N)}=\left|\frac{\mathbf{T}^{(N)}\cdot\mathbf{u}^{(N)}}{\mathbf{T}^{(0)}\cdot\mathbf{u}^{(0)}}\right|=(c^{(N)})^2\left|\frac{\boldsymbol{\psi}^{(N)}\cdot\mathbf{g}^{(N)}}{\boldsymbol{\psi}^{(0)}\cdot\mathbf{g}^{(0)}}\right|,$$

we obtain

$$c_{\mathrm{E}}^{(N)}=(c^{(N)})^2\left|\frac{\mathbf{v}_{\mathrm{g}}^{(N)}\cdot\boldsymbol{\psi}}{\mathbf{v}_{\mathrm{g}}^{(0)}\cdot\boldsymbol{\psi}}\right|$$

for reflected waves ($N=1$, 2 and 3), and

$$c_{\mathrm{E}}^{(N)}=(c^{(N)})^2\frac{\rho^{(2)}}{\rho^{(1)}}\left|\frac{\mathbf{v}_{\mathrm{g}}^{(N)}\cdot\boldsymbol{\psi}}{\mathbf{v}_{\mathrm{g}}^{(0)}\cdot\boldsymbol{\psi}}\right|$$

for transmitted waves ($N=4$, 5 and 6), (6)

with no summation over N. Vectors $\mathbf{v}_{\mathrm{g}}^{(0)}$ and $\mathbf{v}_{\mathrm{g}}^{(N)}$ are the group velocity vectors of the incident and Nth scattered waves.

Instead of using the formulation in eq. (6), some authors (e.g. Chapman 1994) define the energy coefficients by the following formulae:

$$c_{\mathrm{N}}^{(N)}=c^{(N)}\sqrt{\left|\frac{\mathbf{v}_{\mathrm{g}}^{(N)}\cdot\boldsymbol{\psi}}{\mathbf{v}_{\mathrm{g}}^{(0)}\cdot\boldsymbol{\psi}}\right|}$$

for reflected waves ($N=1$, 2 and 3), and

$$c_{\mathrm{N}}^{(N)}=c^{(N)}\sqrt{\frac{\rho^{(2)}}{\rho^{(1)}}\left|\frac{\mathbf{v}_{\mathrm{g}}^{(N)}\cdot\boldsymbol{\psi}}{\mathbf{v}_{\mathrm{g}}^{(0)}\cdot\boldsymbol{\psi}}\right|}$$

for transmitted waves ($N=4$, 5 and 6). (7)

To avoid confusion, we shall refer to these coefficients as to the *normalized R/T coefficients*. Note that unlike the case for the displacement coefficients (5), the energy coefficients (6) and the normalized coefficients (7) are reciprocal.

3 *P*-WAVE INCIDENCE AT A WEAK-CONTRAST INTERFACE BETWEEN ARBITRARY ANISOTROPIC MEDIA

We now consider the P-wave incident at interface Σ. Moreover, we assume that interface Σ separates two half-spaces with very similar anisotropy. In this case, we can express the elasticity parameters $a_{ijkl}^{(I)}$ and density $\rho^{(I)}$ in both half-spaces in terms of the elasticity parameters a_{ijkl}^{0} and density ρ^0 of the background anisotropic medium, which is the same for both half-spaces:

$$a_{ijkl}^{(I)}=a_{ijkl}^{0}+\Delta a_{ijkl}^{(I)},\qquad \rho^{(I)}=\rho^0+\Delta\rho^{(I)},\quad I=1,2. \tag{8}$$

The symbols $\Delta a_{ijkl}^{(I)}$ and $\Delta\rho^{(I)}$ denote the deviations from the parameters of the background medium. We shall require these deviations, or equivalently the *elasticity contrasts* $\Delta a_{ijkl}=a_{ijkl}^{(2)}-a_{ijkl}^{(1)}$ and *density contrast* $\Delta\rho=\rho^{(2)}-\rho^{(1)}$ at the interface, to be small.

Under the weak-contrast condition, we can linearize the vector of the displacement R/T coefficients,

$$\mathbf{c}=(R^{PP}\quad R^{PS1}\quad R^{PS2}\quad T^{PP}\quad T^{PS1}\quad T^{PS2})^{\mathrm{T}},$$

as follows:

$$\mathbf{c}=\mathbf{c}^0+\Delta\mathbf{c}, \tag{9}$$

where $\mathbf{c}^0$ is the vector of R/T coefficients in the background medium and $\Delta\mathbf{c}$ is its small perturbation. Since the background medium is homogeneous, vector $\mathbf{c}^0$ takes the following simple form:

$$\mathbf{c}^0=(0\quad 0\quad 0\quad 1\quad 0\quad 0)^{\mathrm{T}}, \tag{10}$$

expressing the obvious fact that the incident P wave crosses the fictitious (non-existent) interface in the background medium unaffected. For the perturbation

$$\Delta\mathbf{c}=(R^{PP}\quad R^{PS1}\quad R^{PS2}\quad T^{PP}-1\quad T^{PS1}\quad T^{PS2})^{\mathrm{T}},$$

eq. (5) yields (Thomsen 1993, eq. A-18a; Vavryčuk & Pšenčík 1998, eqs 18 and 28)

$$\Delta\mathbf{c}=(\mathbf{D}^0)^{-1}(\Delta\mathbf{d}^{(0)}-\Delta\mathbf{D}\mathbf{c}^0)=-(\mathbf{D}^0)^{-1}(\Delta\mathbf{d}^{(4)}-\Delta\mathbf{d}^{(0)}). \tag{11}$$

The inverse matrix $(\mathbf{D}^0)^{-1}$ can be expressed explicitly (Fryer & Frazer 1984, eq. 3.20; Chapman 1994, eq. 22), and for the perturbation of the R/T coefficient of the Nth wave we finally obtain

$$\boxed{\Delta c^{(N)} = -\frac{\mathbf{d}^{0(N)}\cdot\Delta\mathbf{s}}{\mathbf{d}^{0(N)}\cdot\mathbf{s}^{0(N)}} = -\frac{\mathbf{s}^{0(N)}\cdot\Delta\mathbf{d}}{\mathbf{d}^{0(N)}\cdot\mathbf{s}^{0(N)}}}\,,$$

$$\text{with no summation over } N. \tag{12}$$

Vectors $\mathbf{d}^{0(N)}$ and $\mathbf{s}^{0(N)}$ are the displacement–stress and stress–displacement vectors of the Nth scattered wave in the background medium, and $\Delta\mathbf{s}$ and $\Delta\mathbf{d}$ denote the differences between the stress–displacement and displacement–stress vectors of the transmitted and incident P waves across the interface:

$$\Delta\mathbf{s} = -\begin{bmatrix}\Delta\boldsymbol{\sigma}\\ \Delta\mathbf{g}\end{bmatrix}, \qquad \Delta\mathbf{d} = -\begin{bmatrix}\Delta\mathbf{g}\\ \Delta\boldsymbol{\sigma}\end{bmatrix},$$

$$\Delta\mathbf{g} = \Delta\mathbf{g}^{(4)} - \Delta\mathbf{g}^{(0)}, \qquad \Delta\boldsymbol{\sigma} = \Delta\boldsymbol{\sigma}^{(4)} - \Delta\boldsymbol{\sigma}^{(0)}. \tag{13}$$

In analogy to the terms elasticity and density contrasts, we call the quantities $\Delta\mathbf{s}$, $\Delta\mathbf{d}$, $\Delta\mathbf{g}$ and $\Delta\boldsymbol{\sigma}$ the *stress–displacement*, *displacement–stress*, *polarization* and *traction contrasts.*

Formula (12) is valid for strongly as well as for weakly anisotropic background media and even for isotropic background media. Formulae similar to (12) for the P-wave incidence can also be derived for the $S1$- or $S2$-wave incidences. However, the anisotropy of the background medium must then be sufficiently strong. Our approach cannot be applied to S waves incident at a weak-contrast interface in a weakly anisotropic medium without modification. The reason is the degeneration of the S waves in isotropic media (see Jech & Pšenčík 1989). For the energy and normalized R/T coefficients, eqs (6) and (7) immediately yield

$$c_{\mathrm{E}}^{(N)} = (\Delta c^{(N)})^2\left|\frac{\mathbf{v}_{\mathrm{g}}^{0(N)}\cdot\boldsymbol{\psi}}{\mathbf{v}_{\mathrm{g}}^{0(0)}\cdot\boldsymbol{\psi}}\right|,$$

$$c_{\mathrm{N}}^{(N)} = \Delta c^{(N)}\sqrt{\left|\frac{\mathbf{v}_{\mathrm{g}}^{0(N)}\cdot\boldsymbol{\psi}}{\mathbf{v}_{\mathrm{g}}^{0(0)}\cdot\boldsymbol{\psi}}\right|}, \quad \text{for } N = 1, 2, 3, 5, 6,\ N \neq 4, \tag{14}$$

where $\mathbf{v}_{\mathrm{g}}^{0(0)}$ and $\mathbf{v}_{\mathrm{g}}^{0(N)}$ are the group velocities of the incident and of the Nth scattered waves in the background medium. The formula for the energy coefficient of the transmitted P wave $(N=4)$ is more complicated, and it is more convenient to calculate this coefficient from the energy conservation law for scattered waves:

$$c_{\mathrm{E}}^{(4)} = 1 - c_{\mathrm{E}}^{(1)} - c_{\mathrm{E}}^{(2)} - c_{\mathrm{E}}^{(3)} - c_{\mathrm{E}}^{(5)} - c_{\mathrm{E}}^{(6)}. \tag{15}$$

The normalized coefficient of the transmitted P waves $c_{\mathrm{N}}^{(4)}$, however, is much simpler than the energy coefficient $c_{\mathrm{E}}^{(4)}$, because it equals 1 in first-order perturbation theory. Obviously, the formulae for the energy coefficients are of the order of second perturbations.

4 *P*-WAVE INCIDENCE AT A WEAK-CONTRAST INTERFACE IN WEAKLY ANISOTROPIC MEDIA

In the previous section, we presented a formula for R/T coefficients at weak-contrast interfaces in general anisotropic media, including the case of strong anisotropy. In this section, we will confine ourselves to weakly anisotropic media by assuming that the background medium is isotropic. Under this assumption, we can express the formulae for contrasts $\Delta\mathbf{g}$ and $\Delta\boldsymbol{\sigma}$, and vectors $\mathbf{g}^{0(N)}$ and $\boldsymbol{\sigma}^{0(N)}$ for all scattered waves that are necessary to determine the R/T coefficients from eq. (12). In this section, we will show that contrasts $\Delta\mathbf{g}$ and $\Delta\boldsymbol{\sigma}$ are linear functions of elasticity and density contrasts, Δa_{ijkl} and $\Delta\rho$, in first-order perturbation theory. Obviously, the polarization and traction vectors $\mathbf{g}^{0(N)}$ and $\boldsymbol{\sigma}^{0(N)}$ for unconverted waves do not depend on the elasticity or density contrasts at all. However, the problem is more complicated for converted waves, because in this case polarization and traction vectors $\mathbf{g}^{0(N)}$ and $\boldsymbol{\sigma}^{0(N)}$ are functions of elasticity contrasts Δa_{ijkl}. Although these vectors are specified for the isotropic background medium, they depend on the properties of the perturbed medium (see Jech & Pšenčík 1989). Moreover, this dependence is *non-linear*. Consequently, the R/T coefficients for converted waves are also non-linear functions of elasticity contrasts Δa_{ijkl} at the interface, even in the first-order perturbation theory. In order to avoid this difficulty, we introduce new quantities called *vector R/T coefficients of S waves*, which are defined as follows:

$$\mathbf{R}^{PS} = c^{(2)}\mathbf{g}^{0(2)} + c^{(3)}\mathbf{g}^{0(3)}, \qquad \mathbf{T}^{PS} = c^{(5)}\mathbf{g}^{0(5)} + c^{(6)}\mathbf{g}^{0(6)}. \tag{16}$$

$\mathbf{R}^{PS}$ is called the *vector reflection coefficient*, and $\mathbf{T}^{PS}$ the *vector transmission coefficient of S waves.* Obviously, the scalar R/T coefficient is obtained by projecting the vector coefficient into the direction of polarization of the scattered wave. Later we shall show that the vector R/T coefficients of S waves are linear functions of contrasts Δa_{ijkl}.

In this section we present the formulae for contrasts $\Delta\mathbf{g}$ and $\Delta\boldsymbol{\sigma}$, and vectors $\mathbf{g}^{0(N)}$ and $\boldsymbol{\sigma}^{0(N)}$; in the next section we present the final formulae for the R/T coefficients.

4.1 Polarization and traction contrasts $\Delta\mathbf{g}$ and $\Delta\boldsymbol{\sigma}$

If we perturb the polarization vectors of the incident and transmitted P waves, we obtain the contrast $\Delta\mathbf{g}$ in the following form:

$$\Delta\mathbf{g} = \Delta\mathbf{g}_1 + \Delta\mathbf{g}_2,$$

$$\Delta g_{1m} = \frac{\Delta V}{\alpha}\left(N_m^P - \frac{1}{\cos\theta^P}\nu_m\right),$$

$$\Delta V = \frac{1}{2\alpha}\Delta a_{ijkl}N_i^P N_j^P N_k^P N_l^P, \tag{17}$$

$$\Delta g_{2m} = \frac{\Delta\Gamma_{kl}}{\alpha^2-\beta^2}N_k^P(\delta_{lm} - N_l^P N_m^P), \quad \Delta\Gamma_{kl} = \Delta a_{ijkl}N_i^P N_j^P,$$

$$\cos\theta^P = \mathbf{N}^P\cdot\boldsymbol{\nu},$$

where α and β denote the P- and S-wave velocities in the background isotropic medium, $\mathbf{N}^P$ is the phase normal of the incident and transmitted P waves in the isotropic background medium, and $\boldsymbol{\nu}$ is the normal to the interface. The quantity ΔV is the *P-wave phase-velocity contrast* defined as

$$\Delta V = V^{(4)} - V^{(0)} = \Delta V^{(4)} - \Delta V^{(0)}, \tag{18}$$

where $V^{(0)}$ and $V^{(4)}$ are the phase velocities of the incident and transmitted P waves. The quantity $\Delta\mathbf{g}_1$ denotes the polarization contrast due to the contrast of the phase velocity at the interface and is determined by Snell's law. For no velocity contrast at the interface in weakly anisotropic media, $\Delta\mathbf{g}_1$ is equal to zero. The quantity $\Delta\mathbf{g}_2$ denotes the polarization contrast due to weak anisotropy. For a weak-contrast interface in isotropic media, $\Delta\mathbf{g}_2$ is equal to zero.

Analogously to determining $\Delta\mathbf{g}$, we can perturb the traction vectors of the incident and transmitted P waves to obtain contrast $\Delta\boldsymbol{\sigma}$:

$$\Delta\boldsymbol{\sigma} = \Delta\boldsymbol{\sigma}_1 + \Delta\boldsymbol{\sigma}_2 + \Delta\boldsymbol{\sigma}_3 ,$$

$$\Delta\sigma_{1m} = \frac{\Delta\rho}{\rho}\sigma_m^{0(0)} = \Delta\rho\left[\frac{\alpha^2 - 2\beta^2}{\alpha}\nu_m + 2\frac{\beta^2}{\alpha}\cos\theta^P N_m^P\right],$$

$$\Delta\sigma_{2m} = -\Delta V\frac{\rho}{\cos\theta^P} \times\left\{2\frac{\beta^2}{\alpha^2}\left[\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}\cos^2\theta^P + 1\right]N_m^P + \cos\theta^P\nu_m\right\}, \tag{19}$$

$$\Delta\sigma_{3m} = \Delta\Gamma_{kl}\frac{\rho}{\alpha}\left[\nu_k\delta_{lm} + \frac{\beta^2}{\alpha^2-\beta^2}N_k^P(\cos\theta^P\delta_{lm} + \nu_l N_m^P)\right].$$

$\Delta\boldsymbol{\sigma}_1$ denotes the perturbation due to the density contrast, $\Delta\boldsymbol{\sigma}_2$ is the perturbation due to the P-wave phase-velocity contrast ΔV at the interface, and $\Delta\boldsymbol{\sigma}_3$ is that part of $\Delta\boldsymbol{\sigma}$ that is non-zero even for no density and no velocity contrasts $\Delta\rho$ and ΔV at the interface.

4.2 Polarization and traction vectors $\mathbf{g}^{0(N)}$ and $\boldsymbol{\sigma}^{0(N)}$

For unconverted waves we obtain

$$g_m^{0(1)} = N_m^{RP}, \qquad g_m^{0(4)} = N_m^P,$$

$$\sigma_m^{0(1)} = \frac{\rho}{\alpha}[(\alpha^2 - 2\beta^2)\nu_m + 2\beta^2\cos\theta^{RP}N_m^{RP}],$$

$$\sigma_m^{0(4)} = \frac{\rho}{\alpha}[(\alpha^2 - 2\beta^2)\nu_m + 2\beta^2\cos\theta^{P}N_m^{P}], \tag{20}$$

where $\mathbf{N}^{RP}$ is the phase normal of the reflected P wave in the isotropic background.

As mentioned above, the problem is more complicated for converted waves than for unconverted waves. Both polarization and traction vectors $\mathbf{g}^{0(N)}$ and $\boldsymbol{\sigma}^{0(N)}$ are, for S waves, non-linear functions of the elasticity contrast Δa_{ijkl}. Therefore, the R/T coefficients of S waves are also non-linear functions of Δa_{ijkl}. However, when we determine the *vector* R/T coefficients of S waves from (16), we eliminate this non-linear dependence. The vector R/T coefficients do not depend on vectors $\mathbf{g}^{0(N)}$ and $\boldsymbol{\sigma}^{0(N)}$, but rather on dyadics $\mathbf{g}^{0(N)}\mathbf{g}^{0(N)}$ and $\mathbf{g}^{0(N)}\boldsymbol{\sigma}^{0(N)}$. For reflected S waves we need to express the following terms:

$$g_k^{0(2)}g_l^{0(2)} + g_k^{0(3)}g_l^{0(3)} = \delta_{kl} - N_k^{RS}N_l^{RS},$$

$$\sigma_k^{0(2)}g_l^{0(2)} + \sigma_k^{0(3)}g_l^{0(3)} = \rho\beta(\cos\theta^{RS}\delta_{kl} + N_k^{RS}\nu_l - 2\cos\theta^{RS}N_k^{RS}N_l^{RS}), \tag{21}$$

where

$$\cos\theta^{RS} = \mathbf{N}^{RS}\cdot\boldsymbol{\nu}.$$

$\mathbf{N}^{RS}$ is the phase normal of the reflected S waves in the isotropic background. These formulae are no longer functions of the polarization vectors of S waves, and consequently do not depend on the elasticity contrast Δa_{ijkl} at all. For transmitted S waves we obtain formulae analogous to (21).

5 R/T COEFFICIENTS IN WEAKLY ANISOTROPIC MEDIA

We divide the perturbation of the R/T coefficients $\Delta\mathbf{c}$ into three parts:

$$\Delta\mathbf{c} = \Delta\mathbf{c}_1 + \Delta\mathbf{c}_2 + \Delta\mathbf{c}_3 . \tag{22}$$

The first part, $\Delta\mathbf{c}_1$, is the perturbation due to the density contrast $\Delta\rho$, the second part, $\Delta\mathbf{c}_2$, is caused by the phase-velocity contrast ΔV, and the last part, $\Delta\mathbf{c}_3$, is the part of $\Delta\mathbf{c}$ that can be non-zero even for no density and no phase-velocity contrasts at the interface.

5.1 Displacement coefficients

5.1.1 PP coefficients

Inserting eqs (17), (19) and (20) into (12), for the PP coefficients we obtain

$$R^{PP} = R_1^{PP} + R_2^{PP} + R_3^{PP}, \qquad T^{PP} = 1 + T_1^{PP} + T_2^{PP} + T_3^{PP}, \tag{23}$$

where

$$R_1^{PP} = \frac{1}{2}\frac{\Delta\rho}{\rho}\left[1 - 4\frac{\beta^2}{\alpha^2}\sin^2\theta^P\right], \qquad R_2^{PP} = \frac{1}{2\cos^2\theta^P}\frac{\Delta V}{\alpha},$$

$$R_3^{PP} = \frac{\Delta\Gamma_{kl}}{\alpha^2}\nu_k\left[\nu_l - \frac{1}{\cos\theta^P}N_l^P\right], \tag{24}$$

$$T_1^{PP} = -\frac{1}{2}\frac{\Delta\rho}{\rho}, \qquad T_2^{PP} = \frac{1}{2\cos^2\theta^P}\frac{\Delta V}{\alpha}[1 + 2\cos^2\theta^P],$$

$$T_3^{PP} = -\frac{\Delta\Gamma_{kl}}{\alpha^2}\frac{\nu_k N_l^P}{\cos\theta^P}, \tag{25}$$

and

$$\Delta V = \frac{1}{2\alpha}\Delta a_{ijkl}N_i^P N_j^P N_k^P N_l^P, \qquad \Delta\Gamma_{kl} = \Delta a_{ijkl}N_i^P N_j^P,$$

$$\cos\theta^P = \mathbf{N}^P\cdot\boldsymbol{\nu} < 0 .$$

$\mathbf{N}^P$ is the phase normal of the incident wave in the background medium, $\boldsymbol{\nu}$ is the normal to the interface, θ^P is the angle between the normal $\boldsymbol{\nu}$ and phase normal $\mathbf{N}^P$. α, β and ρ are the P- and S-wave velocities and the density of the isotropic background medium, respectively. Δa_{ijkl} and $\Delta\rho$ are the elasticity and density contrasts at the interface. Note that our definition of the orientations of $\boldsymbol{\nu}$ and $\mathbf{N}^P$ implies that $\cos\theta^P$ is negative.

5.1.2 PS coefficients

The reflection coefficients of converted waves can be calculated from vector coefficient $\mathbf{R}^{PS}$ in the following way:

$$\Delta c^{(2)} = \mathbf{R}^{PS}\cdot\mathbf{g}^{0(2)}, \qquad \Delta c^{(3)} = \mathbf{R}^{PS}\cdot\mathbf{g}^{0(3)}. \tag{26}$$

Inserting eqs (17), (19), (21) and (12) into (16), we can express vector coefficient $\mathbf{R}^{PS}$ as

$$\mathbf{R}^{PS}=\mathbf{R}_1^{PS}+\mathbf{R}_2^{PS}+\mathbf{R}_3^{PS},$$

$$R_{1m}^{PS}=\frac{\Delta\rho}{\rho}\,\frac{1}{2\cos\theta^{RS}}\left\{\frac{\alpha^2-2\beta^2}{\alpha\beta}\,v_m+2\frac{\beta}{\alpha}\cos\theta^P N_m^P\right\},$$

$$\begin{aligned}R_{2m}^{PS}=&-\frac{\Delta V}{\beta}\,\frac{1}{2\cos\theta^P\cos\theta^{RS}}\\&\times\left\{\left[2\frac{\beta^2}{\alpha^2}+\frac{\beta}{\alpha}\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}\cos\theta^P\left(2\frac{\beta}{\alpha}\cos\theta^P+\cos\theta^{RS}\right)\right]N_m^P\right.\\&\left.+\left[\cos\theta^P+\frac{\beta}{\alpha}\left(\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}\cos\theta^P\cos\theta^{PRS}+2\cos\theta^{RS}\right)\right]v_m\right\},\end{aligned}\tag{27}$$

$$\begin{aligned}R_{3m}^{PS}=&\frac{\Delta\Gamma_{kl}}{\alpha^2-\beta^2}\,\frac{1}{2\cos\theta^{RS}}\left\{\frac{\alpha^2-\beta^2}{\alpha\beta}\,v_k\delta_{lm}+N_k^P\right.\\&\left.\times\left[\left(\frac{\beta}{\alpha}\cos\theta^P+\cos\theta^{RS}\right)\delta_{lm}+\frac{\beta}{\alpha}v_l N_m^P+N_l^{RS}v_m\right]\right\},\end{aligned}$$

where

$$\cos\theta^P=\mathbf{N}^P\cdot\mathbf{v}<0,\qquad \cos\theta^{RS}=\mathbf{N}^{RS}\cdot\mathbf{v}>0,$$

$$\cos\theta^{PRS}=\mathbf{N}^P\cdot\mathbf{N}^{RS}.$$

$\mathbf{N}^{RS}$ is the phase normal of the reflected S waves in the background medium (see Fig. 1), θ^{RS} is the angle between phase normal $\mathbf{N}^{RS}$ and the normal to the interface $\mathbf{v}$, and θ^{PRS} is the angle between phase normals $\mathbf{N}^P$ and $\mathbf{N}^{RS}$ (see Fig. 1). Since phase normals $\mathbf{N}^P$ and $\mathbf{N}^{RS}$ are considered for the background medium, angles θ^P and θ^{RS} are related by Snell's law:

$$\sin\theta^{RS}=\frac{\beta}{\alpha}\sin\theta^P.$$

In order to keep the formula for $\mathbf{R}^{PS}$ reasonably short, eq. (27) is only a simplified formula for $\mathbf{R}^{PS}$. This formula is considerably shorter than the complete formula, because we have omitted from (27) all terms that point in the direction of the phase normal $\mathbf{N}^{RS}$. As a consequence, $\mathbf{R}^{PS}$ in (27) is no longer a vector lying in the plane perpendicular to phase normal $\mathbf{N}^{RS}$. Nevertheless, its scalar product with polarization vectors $\mathbf{g}^{0(2)}$ and $\mathbf{g}^{0(3)}$ is unchanged, and thus by calculating the scalar coefficients using eqs (26) and (27) we obtain the correct results. For the determination of the polarization vectors for the reflected S waves, $\mathbf{g}^{0(2)}$ and $\mathbf{g}^{0(3)}$, refer to Jech & Pšenčík (989).

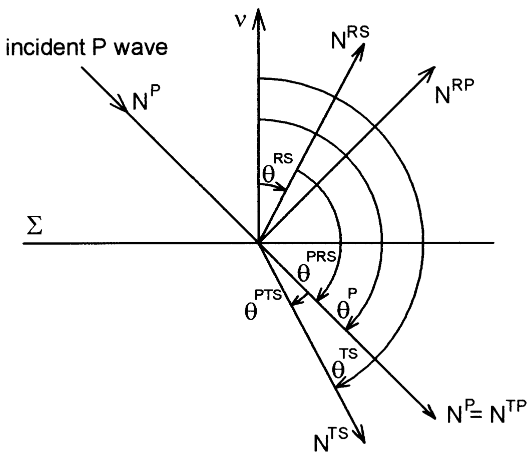

Figure 1. Schematic diagram showing definitions of the phase normals and incidence angles.

The coefficients for the transmitted converted waves can be calculated in a similar way to those for the reflected waves. The formula for $\mathbf{T}^{PS}$ can be obtained from the formula for $\mathbf{R}^{PS}$, if we multiply this formula by -1 and substitute $\mathbf{N}^{TS}$, θ^{TS} and θ^{PTS} for $\mathbf{N}^{RS}$, θ^{RS} and θ^{PRS} (see Fig. 1).

5.2 Energy and normalized coefficients

Using eqs (14) and (15), the energy coefficients can be written as

$$R_{\mathrm{E}}^{PP}=(R^{PP})^2,\quad T_{\mathrm{E}}^{PP}=1-R_{\mathrm{E}}^{PP}-R_{\mathrm{E}}^{PS1}-R_{\mathrm{E}}^{PS2}-T_{\mathrm{E}}^{PS1}-T_{\mathrm{E}}^{PS2},$$

$$R_{\mathrm{E}}^{PS1}=\frac{\beta}{\alpha}\left|\frac{\cos\theta^{RS}}{\cos\theta^P}\right|(R^{PS1})^2,\quad R_{\mathrm{E}}^{PS2}=\frac{\beta}{\alpha}\left|\frac{\cos\theta^{RS}}{\cos\theta^P}\right|(R^{PS2})^2,\tag{28}$$

$$T_{\mathrm{E}}^{PS1}=\frac{\beta}{\alpha}\left|\frac{\cos\theta^{TS}}{\cos\theta^P}\right|(T^{PS1})^2,\quad T_{\mathrm{E}}^{PS2}=\frac{\beta}{\alpha}\left|\frac{\cos\theta^{TS}}{\cos\theta^P}\right|(T^{PS2})^2,$$

and the normalized coefficients as

$$R_{\mathrm{N}}^{PP}=R^{PP},\qquad T_{\mathrm{N}}^{PP}=1,$$

$$R_{\mathrm{N}}^{PS1}=\sqrt{\frac{\beta}{\alpha}\left|\frac{\cos\theta^{RS}}{\cos\theta^P}\right|}\,R^{PS1},\quad R_{\mathrm{N}}^{PS2}=\sqrt{\frac{\beta}{\alpha}\left|\frac{\cos\theta^{RS}}{\cos\theta^P}\right|}\,R^{PS2},\tag{29}$$

$$T_{\mathrm{N}}^{PS1}=\sqrt{\frac{\beta}{\alpha}\left|\frac{\cos\theta^{TS}}{\cos\theta^P}\right|}\,T^{PS1},\quad T_{\mathrm{N}}^{PS2}=\sqrt{\frac{\beta}{\alpha}\left|\frac{\cos\theta^{TS}}{\cos\theta^P}\right|}\,T^{PS2}.$$

5.3 Vertical *P*-wave incidence

For strictly vertical P-wave incidence at the horizontal interface we obtain

$$R^{PP}=\frac{1}{2}\frac{\Delta\rho}{\rho}+\frac{1}{2}\frac{\Delta V}{\alpha},$$

$$R_1^{PS}=\frac{1}{2\beta}\frac{\Delta a_{35}}{\alpha+\beta},\qquad R_2^{PS}=\frac{1}{2\beta}\frac{\Delta a_{34}}{\alpha+\beta},\qquad R_3^{PS}=0,$$

$$T^{PP}=1-R^{PP},$$

$$T_1^{PS}=\frac{1}{2\beta}\frac{\Delta a_{35}}{\alpha-\beta},\qquad T_2^{PS}=\frac{1}{2\beta}\frac{\Delta a_{34}}{\alpha-\beta},\qquad T_3^{PS}=0,\tag{30}$$

where

$$\Delta V=\frac{1}{2\alpha}\Delta a_{33}.$$

Note that for no density contrast and no P-wave phase-velocity contrast, we observe no reflected P wave, but we can observe reflected S waves. The reflected S waves will be observed even if the half-space in which the incident P wave propagates is isotropic. This is of particular interest because such a phenomenon is unknown for interfaces between two isotropic media. This means, for example, that the detection of reflected S waves for a vertical incident P wave can serve as clear evidence for the presence of anisotropic structures beneath isotropic subsurface layers, because in this case no contrast between two horizontal isotropic layers could generate the reflected S wave.

6 NUMERICAL EXAMPLE

6.1 Forward modelling

The accuracy of the perturbation formulae for the *PP* displacement R/T coefficients in weakly anisotropic media has been extensively tested by Pšenčík & Vavryčuk (1998) and by Vavryčuk & Pšenčík (1998). The authors show that the perturbation formulae for the *PP* R/T coefficients are reasonably accurate not only under weak anisotropy and for weak-contrast interfaces, but also in cases where the contrast at the interface and/or anisotropy of the half-spaces is fairly strong (nearly 20 per cent). Therefore, we shall not repeat the numerical tests for the *PP* waves here, but confine ourselves to numerical tests on the approximate formulae for the *PS* coefficients.

We shall examine the behaviour of the *PS* reflection coefficient for two models used by Vavryčuk & Pšenčík (1998), referred to as model A/C and model A/D. The models consist of homogeneous half-spaces separated by a horizontal interface. The incident *P* wave propagates in the upper half-space (medium A), which is isotropic. The *P*- and *S*-wave velocities and the density of the half-space are $V_{1P}=4.00\ \mathrm{km\ s^{-1}}$, $V_{1S}=2.31\ \mathrm{km\ s^{-1}}$ and $\rho_1=2.65\ \mathrm{g\ cm^{-3}}$. The lower half-space is transversely isotropic with a horizontal axis of symmetry. The anisotropy of the lower half-space is assumed to be caused by a system of vertical parallel penny-shaped dry cracks (Hudson 1981). The *P*- and *S*-wave velocities and density of the host rock are $V_{P\text{-}ROCK}=4.00\ \mathrm{km\ s^{-1}}$, $V_{S\text{-}ROCK}=2.31\ \mathrm{km\ s^{-1}}$ and $\rho_2=2.60\ \mathrm{g\ cm^{-3}}$. The crack normal is parallel to the x-axis, and the crack density is either $e=0.05$ (medium C) or $e=0.1$ (medium D). The corresponding density-normalized elastic parameters in Voigt notation are $a_{11}=11.96$, $a_{13}=3.99$, $a_{33}=15.55$, $a_{44}=5.33$, and $a_{66}=4.76$ for crack density $e=0.05$; and $a_{11}=9.43$, $a_{13}=3.14$, $a_{33}=15.27$, $a_{44}=5.33$, and $a_{66}=4.25$ for crack density $e=0.1$. The phase velocities in the x-z plane as a function of the incidence angle for both models are shown in Fig. 2. Anisotropies of the *P*, *S*1 and *S*2 waves reach values of 13.1, 5.7 and 0.4 per cent for medium C, and 23.9, 11.3 and 0.9 per cent for medium D. To calculate the approximate reflection coefficients we have used the following parameters of the isotropic background: $\alpha=3.97\ \mathrm{km\ s^{-1}}$, $\beta=2.25\ \mathrm{km\ s^{-1}}$ and $\rho=2.63\ \mathrm{g\ cm^{-3}}$ for model A/C; and $\alpha=3.95\ \mathrm{km\ s^{-1}}$, $\beta=2.19\ \mathrm{km\ s^{-1}}$ and $\rho=2.63\ \mathrm{g\ cm^{-3}}$ for model A/D. These values were obtained by averaging the densities and vertical velocities in both the half-spaces (see Vavryčuk & Pšenčík 1998).

Similarly to the reflection coefficients of *P* waves (Vavryčuk & Pšenčík 1998), approximate reflection coefficients for *S* waves start to deviate considerably from exact coefficients for

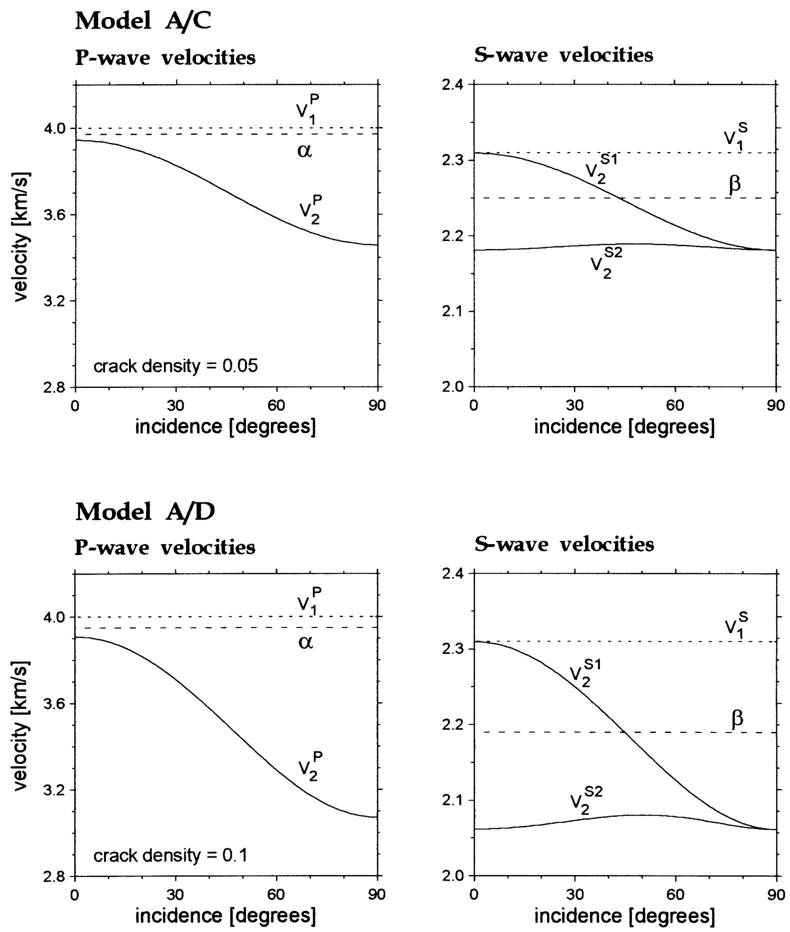

Figure 2. *P*- and *S*-wave phase velocities in the x-z plane. The incidence angle is defined as the angle between the phase normal of the wave and the z-axis. α and β are the *P*- and *S*-wave velocities of the isotropic background (for values see the text). v_1^P and v_1^S are *P*- and *S*-wave velocities in the upper isotropic half-space; v_2^P, v_2^{S1}, and v_2^{S2} are the *P*- and *S*-wave velocities in the lower anisotropic half-space.

larger values of the angle of incidence, and hence the angles of incidence considered have been restricted to the range (0°, 40°). The reflection coefficients for *S* waves are displayed for each model in the form of six plots (see Figs 3 and 4). In all the plots, the horizontal axis corresponds to the angle of incidence θ^P, and the vertical axis corresponds to the azimuth φ. Azimuth $\varphi = 0°$ corresponds to the profile in the *x*-*z* plane. The upper plots show the exact displacement *PSV* reflection coefficient and the relative errors of the approximate displacement and energy coefficients. The lower plots show the exact displacement *PSH* reflection coefficient and the relative errors of the approximate displacement and energy coefficients. The exact energy *PSV* and *PSH* reflection coefficients are not shown, since their directional dependence displays a pattern similar to that of the displacement coefficients.

Figs 3 and 4 indicate that the relative errors of the approximate displacement reflection coefficient for the *SV* waves behave in a way similar to that of the errors of the displacement reflection coefficient for the *P* waves (see Vavryčuk & Pšenčík 1998, Figs 2 and 3). The errors are very similar in directional dependence as well as in magnitude. The relative error of the displacement coefficient is always less than 8 per cent for angles of incidence up to 30° in the A/C model and less than 13 per cent in the A/D model. Furthermore, the approximate displacement reflection coefficient for the *SH* waves displays an accuracy similar to that of the *SV* waves, although its directional dependence is quite different. For azimuths between 80° and 90° the formula for the *SH* waves does not work as well, but the accuracy is still acceptable. Thus we can conclude that the numerical test has proved that the performance of the approximate formulae for the displacement coefficients is good. However, the approximate formulae for the energy coefficients are considerably worse: the relative errors of the energy coefficients are about twice as large as for the displacement coefficients.

6.2 Inverse modelling

We now give a simple example of the use of the approximate formulae for R/T coefficients in the inverse problem. We assume that the exact values of the displacement *PP* reflection coefficient $R^{PP}(\theta^P, \varphi)$ are known for a range of azimuths and

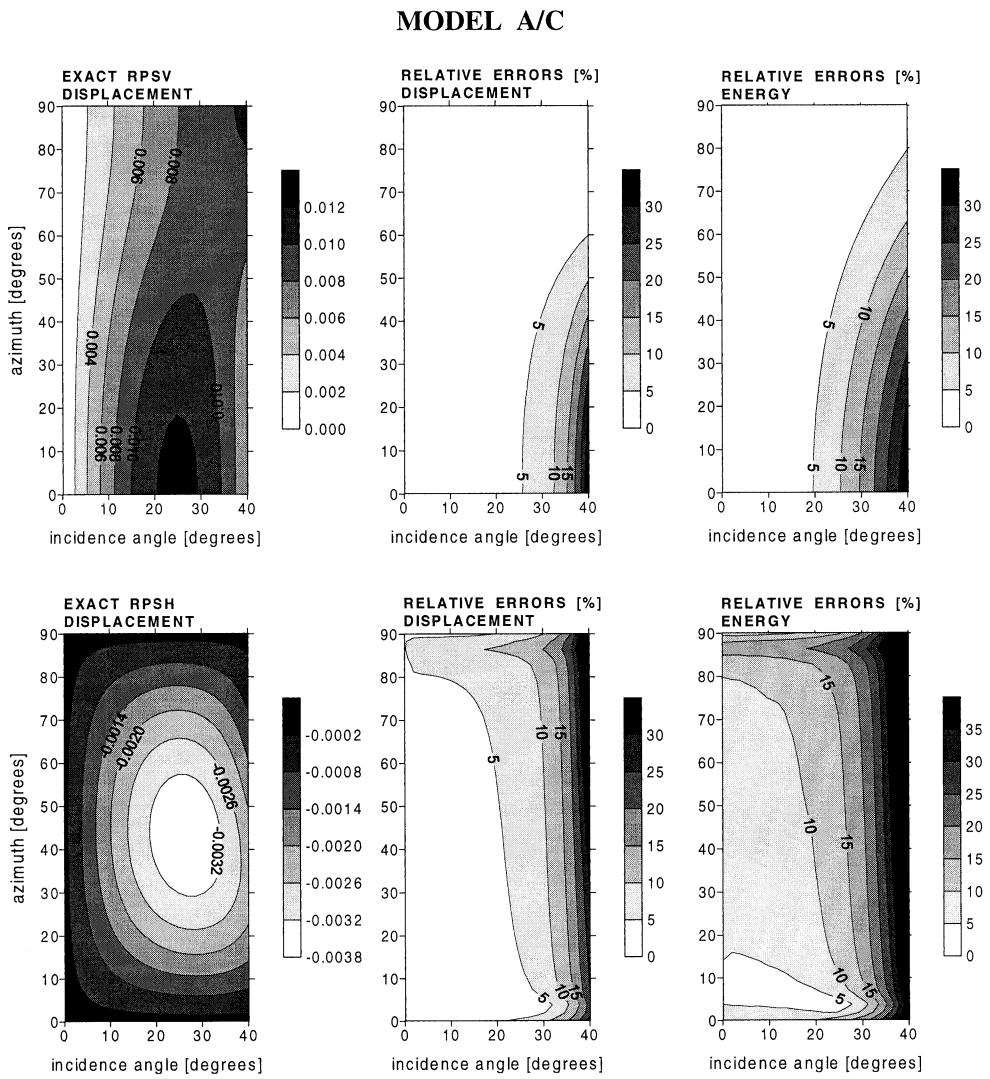

Figure 3. Exact displacement reflection coefficients for *SV* and *SH* waves together with the relative errors of the approximate displacement and energy coefficients for model A/C.

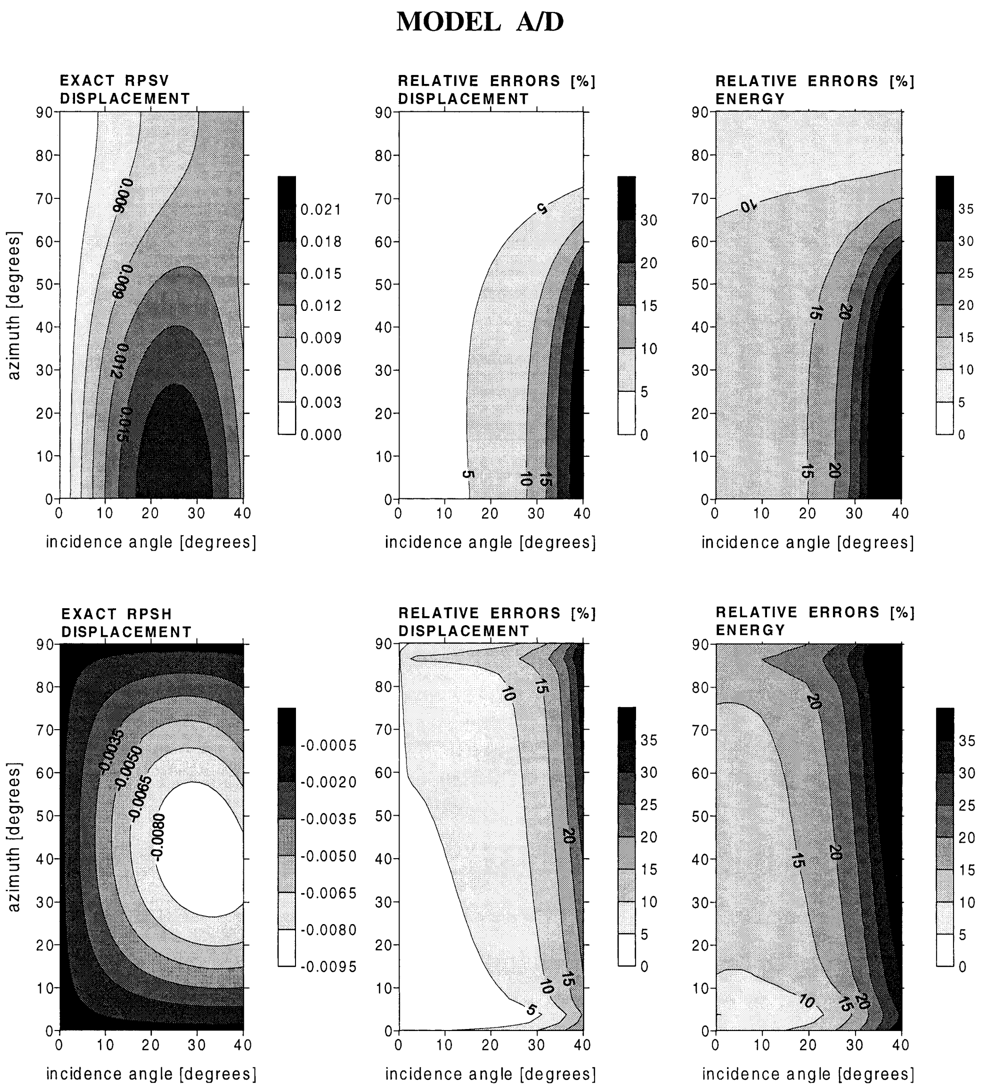

Figure 4. Exact displacement reflection coefficients for SV and SH waves together with the relative errors of the approximate displacement and energy coefficients for model A/D.

incidence angles in the models used in the previous section, and we try to invert them for elasticity and density contrasts Δa_{ijkl} and $\Delta\rho$ at the interface. The inversion is performed with the help of formulae (23) and (24) for the approximate *PP* reflection coefficient. For simplicity, we assume that the upper half-space is isotropic and that the lower half-space is transversely isotropic with a horizontal axis of symmetry. Thus we invert for the following six unknown parameters: Δa_{11}, Δa_{33}, Δa_{13}, Δa_{44}, Δa_{66} and $\Delta\rho$. Since the approximate coefficient $R^{PP}(\theta^P, \varphi)$ depends on the elasticity and density contrasts linearly, the partial derivatives of $R^{PP}(\theta^P, \varphi)$ with respect to the elasticity and density contrasts, $\Delta R^{PP}/\Delta a_{11}$, $\Delta R^{PP}/\Delta a_{33}$, $\Delta R^{PP}/\Delta a_{13}$, $\Delta R^{PP}/\Delta a_{44}$, $\Delta R^{PP}/\Delta a_{66}$ and $\Delta R^{PP}/\Delta\rho$, are mutually independent. All these partial derivatives (see Fig. 5) depend on the azimuth φ and incidence angle θ^P, except for the function $\Delta R^{PP}/\Delta\rho$, which depends only on the incidence angle θ^P.

In the inversion routine, we apply the Levenberg–Marquardt least-squares method (Press *et al.* 1992). We perform the inversion with values of coefficient $R^{PP}(\theta^P, \varphi)$ measured for three sets of angles θ^P and φ. We invert the values of R^{PP} in the range of azimuths $0^\circ \le \varphi \le 90^\circ$, and in three different ranges of the incidence angle: $0^\circ \le \theta^P \le 25^\circ$, $0^\circ \le \theta^P \le 20^\circ$ and $0^\circ \le \theta^P \le 15^\circ$. The R^{PP} coefficient is specified in the above-mentioned intervals in steps of 5°. Thus the elasticity and density contrasts are retrieved from 96, 77 or 58 values of the exact R^{PP} coefficient, respectively.

Tables 1 and 2 display the results of the inversion for models A/C and A/D. The inversion is successful for both models. As expected, the inversion produces better results for the A/C than for the A/D model. For the A/C model, the weak-contrast condition is satisfied better, and the linearized formulae (23) and (24) approximate the R^{PP} coefficient better (see Figs 3 and 4). If we compare the results of the inversion for different data sets, we arrive at a similar conclusion. For both models, the highest accuracy of the retrieved parameters is obtained when inverting values of the R^{PP} coefficient from nearly vertical directions, because in this case the approximate formulae work optimally. This can be seen clearly in Fig. 6, which displays the relative errors of the phase velocities in the lower half-space calculated from the elastic parameters of the isotropic upper half-space and from the retrieved contrasts at the interface. Note that the accuracy of the retrieved parameters

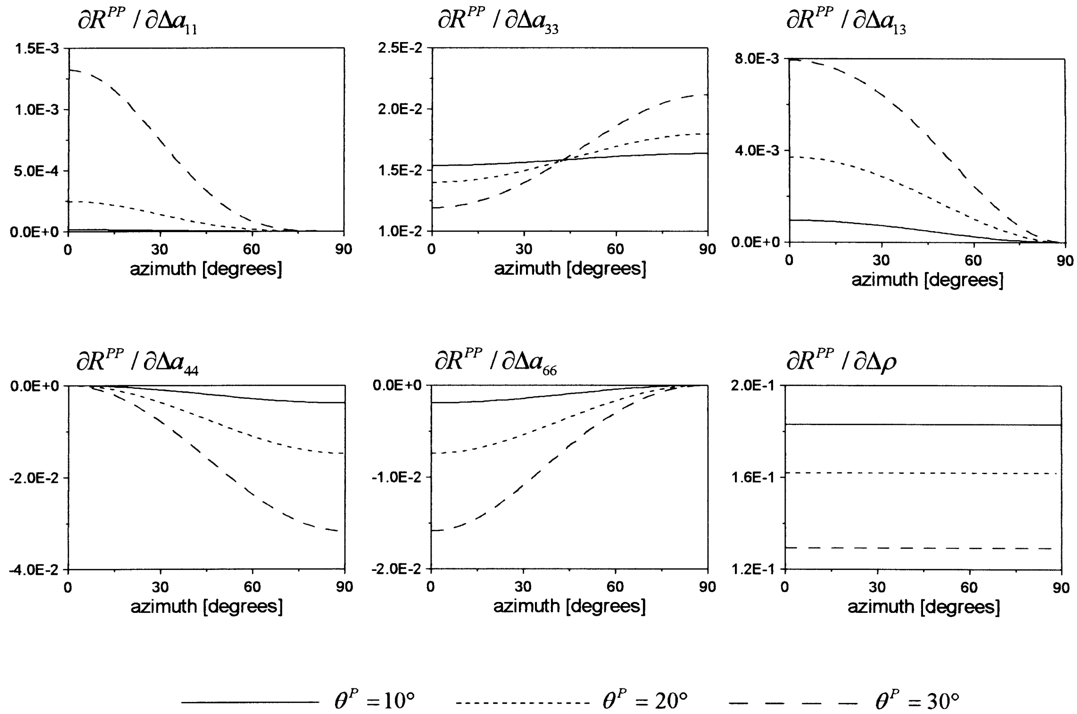

Figure 5. Partial derivatives of the approximate displacement *PP* reflection coefficient with respect to elasticity and density contrasts. The partial derivatives are shown as a function of azimuth φ and for selected values of incidence angle θ^P: $\theta^P = 10°$, 20° and 30°.

Table 1. Inverted elasticity and density contrasts for the A/C model.

Model A/C	Δa_{11}	Δa_{33}	Δa_{13}	Δa_{44}	Δa_{66}	$\Delta\rho$
$\theta^P \leq 25°$	−3.56	−0.44	−1.21	0.00	−0.54	−0.05
$\theta^P \leq 20°$	−3.62	−0.44	−1.21	0.00	−0.55	−0.05
$\theta^P \leq 15°$	−3.66	−0.45	−1.21	−0.01	−0.55	−0.05
exact values	−4.04	−0.45	−1.35	0.00	−0.58	−0.05

Table 2. Inverted elasticity and density contrasts for the A/D model.

Model A/D	Δa_{11}	Δa_{33}	Δa_{13}	Δa_{44}	Δa_{66}	$\Delta\rho$
$\theta^P \leq 25°$	−5.34	−0.70	−1.77	0.00	−1.00	−0.05
$\theta^P \leq 20°$	−5.49	−0.71	−1.78	0.00	−1.01	−0.05
$\theta^P \leq 15°$	−5.61	−0.73	−1.78	−0.01	−1.00	−0.05
exact values	−6.57	−0.73	−2.19	0.00	−1.08	−0.05

does not increase if the interval of incidence angles θ^P is narrower than $\langle 0°, 15° \rangle$, because the number of values of the R^{PP} coefficient to be inverted is then fairly small and the inversion becomes unstable.

7 CONCLUSIONS

We have derived the approximate formulae for the displacement and energy R/T coefficients of *PP* and *PS* plane waves at weak-contrast interfaces in weakly anisotropic elastic media. The formulae are expressed in a coordinate system, which is not, in general, connected with the interface and the incidence plane. Therefore, the formulae are valid for arbitrary orientations of the incidence plane and interface without the need to transform the elasticity parameters into a local Cartesian coordinate system. The formulae for the *PP* R/T coefficients are linear functions of elasticity and density contrasts Δa_{ijkl} and $\Delta\rho$ at the interface. The formulae for the *PS* R/T coefficients are, however, non-linear functions of contrasts Δa_{ijkl}. We therefore introduced new quantities called the *vector R/T coefficients of S waves* or the *vector PS R/T coefficients*. These coefficients are linear functions of contrasts Δa_{ijkl} and $\Delta\rho$, similar to the R/T coefficients of the *PP* waves. The scalar *PS* R/T coefficients can be easily calculated from the vector R/T coefficients and polarization directions of scattered *S* waves. The accuracy of the approximate formulae derived has been numerically tested for the reflection coefficients of *S* waves for models containing a horizontal interface between an isotropic half-space and a transversely isotropic half-space with the symmetry axis lying in the interface. The transverse isotropy is caused by the presence of cracks with crack density $e = 0.05$ (model A/C), or $e = 0.1$ (model A/D). The comparison of the exact and approximate displacement coefficients shows that the approximate formulae are sufficiently accurate for all azimuths and for incidence angles up to 35°. The formulae for the energy coefficients display the worst accuracy, being applicable for incidence angles only up to 20°–25°. For the above-mentioned models, we have also tested the possibility of using the approximate formulae in the inverse problem. With knowledge of the values of the exact displacement *PP* reflection coefficient for a range of azimuths and incidence angles, and using the approximate formula for the *PP* coefficient in the inversion routine, we succeeded in retrieving the elasticity and density contrasts at the interface. The error in the phase velocities of the lower half-space was less than 2 per cent for the A/C model and less than 6 per cent for the A/D model. Note that the *P*-wave anisotropy of the lower half-space in the A/C model reaches 13.1 per cent, and in the A/D model almost 23.9 per cent. Such values of anisotropy are fairly high compared with the anisotropy detected in real seismic structures (Thomsen 1986).

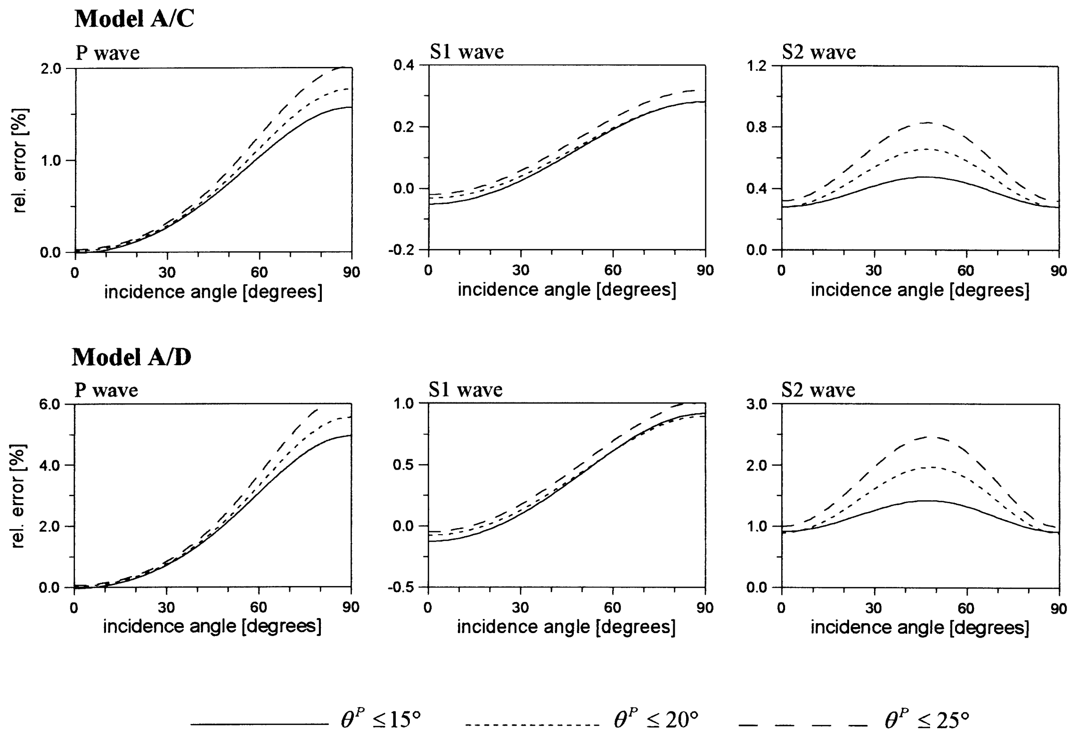

Figure 6. Relative errors of the P-, $S1$- and $S2$-wave phase velocities of the lower half-space retrieved from the inversion of the R^{PP} coefficient.

Therefore, we expect the formulae to yield relevant results in most practical applications in which near-vertical incidences ($\theta^P < 30°$) are considered.

ACKNOWLEDGMENTS

I thank V. Červený and I. Pšenčík for many valuable discussions on the subject and for their comments. I thank two anonymous reviewers for their stimulating reviews. The work was supported by the Grant Agency of the Czech Republic, Grant No. 205/96/0968, by the EC INCO-Copernicus Project IC15 CT96 200, and by the Consortium Project 'Seismic waves in complex 3-D structures'.

REFERENCES

Chapman, C.H., 1994. Reflection/transmission coefficient reciprocities in anisotropic media, *Geophys. J. Int.,* **116,** 498–501.

Daley, P.F. & Hron, F., 1977. Reflection and transmission coefficients for transversely isotropic media, *Bull. seism. Soc. Am.,* **67,** 661–675.

Fryer, G.J. & Frazer, L.N., 1984. Seismic waves in stratified anisotropic media, *Geophys. J. R. astr. Soc.,* **78,** 691–710.

Gajewski, D. & Pšenčík, I., 1987. Computation of high-frequency seismic wavefields in 3-D laterally inhomogeneous anisotropic media, *Geophys. J. R. astr. Soc.,* **91,** 383–411.

Haugen, G.U. & Ursin, H., 1996. AVO-A analysis of vertically fractured reservoir underlaying shale, in *66th Ann. Int. Mtg SEG,* Expanded Abstracts, pp. 1826–1829, SEG, Tulsa.

Hudson, J.A., 1981. Wave speeds and attenuation of elastic waves in material containing cracks, *Geophys. J. R. astr. Soc.,* **64,** 133–150.

Jech, J. & Pšenčík, I., 1989. First-order perturbation method for anisotropic media, *Geophys. J. Int.,* **99,** 369–376.

Keith, C.M. & Crampin, S., 1977. Seismic body waves in anisotropic media: Reflection and refraction at a plane interface, *Geophys. J. R. astr. Soc.,* **49,** 181–208.

Press, W.H., Teukolsky, S.A., Vetterling, W.T. & Flannery, B.P., 1992. *Numerical Recipes. The Art of Scientific Computing,* Cambridge University Press, Cambridge.

Pšenčík, I. & Vavryčuk, V., 1998. Weak contrast *PP* wave displacement R/T coefficients in weakly anisotropic elastic media, *Pure appl. Geophys.,* **151,** 699–718.

Rueger, A., 1997. P-wave reflection coefficients for transversely isotropic models with vertical and horizontal axes of symmetry, *Geophysics,* **62,** 713–722.

Thomsen, L., 1986. Weak elastic anisotropy, *Geophysics,* **51,** 1954–1966.

Thomsen, L., 1993. Weak anisotropic reflections, in *Offset-Dependent Reflectivity–Theory and Practice of AVO Analysis,* SEG Investigations in Geophysics Vol. 8, pp. 103–111, eds Castagna, J.P. & Backus, M.M., SEG, Tulsa.

Vavryčuk, V. & Pšenčík, I., 1998. *PP* wave reflection coefficients in weakly anisotropic elastic media, *Geophysics,* **63,** 2129–2141.

Zillmer, M., Gajewski, D. & Kashtan, B.M., 1998. Anisotropic reflection coefficients for a weak contrast interface, *Geophys. J. Int.,* **132,** 159–166.