Focal mechanisms in anisotropic media

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SUMMARY

Focal mechanisms of seismic sources in anisotropic media are more complicated than in isotropic media. Planar shear faulting produces pure double-couple (DC) mechanism in isotropy, but generally non-double-couple (non-DC) mechanism in anisotropy. The non-DC mechanism can comprise both the isotropic (ISO) and compensated linear vector dipole (CLVD) components. The amount of the ISO and CLVD components depends on strength and symmetry of anisotropy and on the orientation of faulting. Shear faulting in anisotropy generates a pure DC mechanism, provided that faulting is situated in symmetry planes of orthorhombic or of higher anisotropy symmetries. The fault plane solution, i.e. the orientation of the fault normal and the slip direction, can be retrieved from the seismic moment tensor and from elastic parameters describing anisotropy at the source. Numerical modelling shows that shear faulting in anisotropic rocks present in the Earth crust, the Earth mantle or in the subduction zones can produce mostly mechanisms with the CLVD up to 30 per cent and with the ISO up to 15 per cent. The fault plane solutions calculated under the assumption of isotropy typically deviate from the true solutions by an angle of 5° to 10°.

Key words: anisotropy, earthquake, focal mechanism, moment tensor, shear faulting.

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1 INTRODUCTION

Anisotropy is a pervasive property of geological structures in the Earth crust and the upper mantle (Babuška & Cara 1991; Rabbel & Mooney 1996; Silver 1996; Savage 1999). It may be caused by sediment layering, stress-aligned systems of microcracks, cracks or fractures, or the textural ordering of rock-forming minerals. Anisotropy significantly affects seismic observations: it affects the propagation of seismic waves as well as the generation of waves by seismic sources. So far seismologists have focused mainly on studying the theoretical aspects of wave propagation in anisotropic media (e.g. Musgrave 1970; Helbig 1994; Červený 2001) and on observing the effects of anisotropy on seismic waves (Babuška & Cara 1991) such as, for example, the directional variation of seismic velocities or shear wave splitting, detected and measured in situ (Kaneshima et al. 1988; Crampin 1993; Savage 1999) or in the laboratory on rock samples (Kern & Schmidt 1990; Pros et al. 1998; Mainprice et al. 2000).

However, equally important is the way in which anisotropy affects the generation of seismic waves. This comprises the problem of calculating the Green functions (Burridge 1967; Ben-Menahem & Sena 1990a,b; Ben-Menahem *et al.* 1991; Gajewski 1993; Vavryčuk 1997; Pšenčík 1998; Červený 2001), and the problem of calculating seismic moment tensors and focal mechanisms in anisotropic media. As regards the focal mechanisms, Kawasaki & Tanimoto (1981) pointed out that shear faulting in anisotropic media can produce mechanisms with non-double-couple (non-DC) components and nodal lines with peculiarities not observed in isotropic media. Because the non-DC mechanisms of earthquakes are frequently observed (Sipkin 1986; Kuge & Kawakatsu 1992; Kuge & Lay 1994; Miller et al. 1998), the problem, whether anisotropy contributes to them or not, is not only of theoretical interest, but also of practical relevance. Anisotropy as a possible cause of non-DC mechanisms has been mentioned and discussed also by other authors (Kawakatsu 1991; Frohlich 1994; Julian et al. 1998; Vavryčuk 2002; Rössler et al. 2003; Vavryčuk 2004). Because the effects of anisotropy on focal mechanisms are still not well understood, we present a detailed mathematical description of the problem. We define conditions under which shear faulting in anisotropic media can be represented by a pure double couple (DC) and describe the procedure of determining the fault plane solutions in anisotropic media. In numerical modelling, we consider various anisotropy models and illustrate properties of the non-DC mechanisms in dependence on strength and symmetry of anisotropy and on the orientation of faulting. We also estimate errors in determining nodal planes as a result of neglecting anisotropy.

2 MOMENT TENSORS

2.1 Isotropic medium

The moment tensor \mathbf{M} of a seismic source in an isotropic medium is expressed as (Aki & Richards 2002, eq. 3.21)

$$M_{ij} = uS[\lambda v_k n_k \delta_{ij} + \mu(v_i n_j + v_j n_i)], \qquad (1)$$

where *u* is the slip, *S* is the fault area, λ and μ are the Lamé constants describing the isotropic medium surrounding the fault, ν is the slip direction and **n** is the fault normal. For shear sources ($\nu \perp \mathbf{n}$), eq. (1) further simplifies to (Aki & Richards 2002, eq. 3.22)

$$M_{ij} = \mu u S(\nu_i n_j + \nu_j n_i), \tag{2}$$

which is the standard DC representation of an earthquake source.

2.2 Anisotropic medium

The moment tensor **M** of a seismic source in an anisotropic medium is expressed as (Aki & Richards 2002, eq. 3.19)

$$M_{ij} = u S c_{ijkl} v_k n_l, \tag{3}$$

where c_{ijkl} are the elastic parameters of the anisotropic medium surrounding the fault. Introducing tensor **D** and taking into account the symmetry of c_{ijkl} , we can express eq. (3) in the following simple form:

$$M_{ij} = c_{ijkl} D_{kl}.$$
(4)

Tensor **D** denotes the symmetric dyadic tensor

$$D_{kl} = \frac{uS}{2}(v_k n_l + v_l n_k) \tag{5}$$

called hereinafter the seismic source tensor or shortly the source tensor. Eq. (4) is similar to the generalized Hooke's law, but instead of the relation between the stress and strain tensors, it expresses the relation between the moment and source tensors. Therefore, it will be called the generalized Hooke's law at the source. Eq. (4) can equivalently be expressed in matrix form as

$$\mathbf{m} = \mathbf{C}\mathbf{d},\tag{6}$$

where **C** is the 6×6 matrix of the elastic parameters in the two-index Voigt notation, where the subscripts are substituted in the following way: $11 \rightarrow 1$, $22 \rightarrow 2$, $33 \rightarrow 3$, $23 \rightarrow 4$, $13 \rightarrow 5$ and $12 \rightarrow 6$ (see Musgrave 1970, eq. 3.13.4). Quantities **m** and **d** are the 6-vectors defined as

$$\mathbf{m} = (M_{11}, M_{22}, M_{33}, M_{23}, M_{13}, M_{12})^T,$$
(7)

 $\mathbf{d} = uS(n_1v_1, n_2v_2, n_3v_3, n_2v_3)$

$$+ n_3 \nu_2, n_1 \nu_3 + n_3 \nu_1, n_1 \nu_2 + n_2 \nu_1)^T.$$
 (8)

Source tensor **D** is expressed by the components of vector **d** as follows:

$$\mathbf{D} = \frac{1}{2} \begin{bmatrix} 2d_1 & d_6 & d_5 \\ d_6 & 2d_2 & d_4 \\ d_5 & d_4 & 2d_3 \end{bmatrix}.$$
 (9)

2.3 Non-DC components

The moment tensor of a source in anisotropic media has a more general form than in isotropic media. It comprises the DC, the isotropic (ISO) and the compensated linear vector dipole (CLVD) parts (Knopoff & Randall 1970; Jost & Hermann 1989; Lay & Wallace 1995; Vavryčuk 2001):

$$\mathbf{M} = \mathbf{M}^{\mathrm{ISO}} + \mathbf{M}^{\mathrm{CLVD}} + \mathbf{M}^{\mathrm{DC}},\tag{10}$$

where

$$\mathbf{M}^{\rm ISO} = \frac{1}{3} \operatorname{Trace} (\mathbf{M}) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
(11)

$$\mathbf{M}^{\text{CLVD}} = |\varepsilon| M^*_{|\text{MAX}|} \begin{bmatrix} -1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 2 \end{bmatrix},$$
(12)

$$\mathbf{M}^{\rm DC} = (1 - 2|\varepsilon|) M^*_{|\rm MAX|} \begin{bmatrix} -1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1 \end{bmatrix}.$$
 (13)

Parameter ε measures the size of CLVD relative to DC (Sipkin 1986; Kuge & Lay 1994; Julian *et al.* 1998, eq. 18) and is defined as

$$\varepsilon = -\frac{M_{|\text{MIN}|}^*}{\left|M_{|\text{MAX}|}^*\right|},\tag{14}$$

where $M^*_{|MAX|}$ and $M^*_{|MIN|}$ are the eigenvalues of deviatoric moment $\mathbf{M}^* = \mathbf{M}^{\text{CLVD}} + \mathbf{M}^{\text{DC}}$ with the maximum and minimum absolute values, respectively. To assess relative amounts of the DC, CLVD and ISO components in a moment tensor, we usually calculate their percentages:

$$ISO = \frac{1}{3} \frac{\text{Trace}(\mathbf{M})}{|M_{|\text{MAX}|}|} \cdot 100 \text{ per cent},$$
(15)

$$CLVD = 2\varepsilon(100 \text{ per cent} - |ISO|), \qquad (16)$$

$$DC = 100 \text{ per cent} - |ISO| - |CLVD|, \qquad (17)$$

where $M_{|MAX|}$ denotes that eigenvalue of **M**, which has the maximum absolute value. The DC component is always positive; the isotropic and CLVD components can be positive or negative. The sum of the ISO and CLVD components is called the non-DC component of **M**. The sum of the absolute values of the DC and non-DC components is 100 per cent. The non-DC component is zero for shear faulting in isotropic media but generally non-zero for shear faulting in anisotropic media (Kawasaki & Tanimoto 1981).

2.4 DC moment tensors in anisotropic media

Under special geometry of faulting and special types of anisotropy, the moment tensor of a source in anisotropic media can simplify and have only the DC component. Assuming, for example, shear faulting with $\mathbf{n} = (0, 0, 1)^T$ and $\boldsymbol{\nu} = (1, 0, 0)^T$ in triclinic anisotropy, eq. (6) yields

$$\mathbf{m} = \mathbf{C}\mathbf{d} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{33} & C_{34} & C_{35} & C_{36} \\ & & C_{44} & C_{45} & C_{46} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ uS \\ 0 \end{bmatrix}$$
$$= uS \begin{bmatrix} C_{15} \\ C_{25} \\ C_{35} \\ C_{45} \\ C_{55} \\ C_{56} \end{bmatrix}.$$
(18)

Hence, the moment tensor is expressed as

$$\mathbf{M} = uS \begin{bmatrix} C_{15} & C_{56} & C_{55} \\ & C_{25} & C_{45} \\ & & C_{35} \end{bmatrix}.$$
 (19)

The condition for the pure DC moment tensor implies that $Trace(\mathbf{M}) = 0$ and $Det(\mathbf{M}) = 0$. The first condition is the condition for zero ISO and the second condition ensures zero CLVD.

Both conditions are satisfied, for example, for $C_{15} = C_{25} = C_{35} = 0$. Therefore, if anisotropy is orthorhombic or of higher symmetry, the moment tensor must be pure DC:

$$\mathbf{m} = \mathbf{C}\mathbf{d} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ uS \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ M_0 \\ 0 \end{bmatrix}, \qquad (20)$$

where $M_0 = uSC_{55}$ is the scalar moment. Similar results are obtained also for **n** and ν lying along the other symmetry axes, but the couples are scaled by different scalar moments:

$$\mathbf{M} = M_0 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad M_0 = uSC_{66},$$

for $\mathbf{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\boldsymbol{\nu} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$ (21)

$$\mathbf{M} = M_0 \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad M_0 = uSC_{55},$$

for $\mathbf{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\boldsymbol{\nu} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$ (22)

$$\mathbf{M} = M_0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad M_0 = uSC_{44},$$

for $\mathbf{n} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\boldsymbol{\nu} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$ (23)

If fault normal **n** coincides with a symmetry axis and slip ν lies in a symmetry plane (but not along a symmetry axis), then **M** is also pure DC. However, the DC can deviate from the plane defined by the fault normal and slip direction.

2.5 Zero-trace moment tensors in anisotropic media

We can also derive conditions imposed on elastic parameters, under which shear faulting in an anisotropic medium generates no ISO component. This happens if the trace of **M** is zero. Eqs (6)–(8) yield the following conditions under triclinic anisotropy:

$$C_{16} + C_{26} + C_{36} = 0$$
, for $\mathbf{n} = \begin{bmatrix} 1\\0\\0 \end{bmatrix}$ and $\boldsymbol{\nu} = \begin{bmatrix} 0\\1\\0 \end{bmatrix}$, (24)

$$C_{15} + C_{25} + C_{35} = 0$$
, for $\mathbf{n} = \begin{bmatrix} 1\\0\\0 \end{bmatrix}$ and $\boldsymbol{\nu} = \begin{bmatrix} 0\\0\\1 \end{bmatrix}$, (25)

$$C_{14} + C_{24} + C_{34} = 0$$
, for $\mathbf{n} = \begin{bmatrix} 0\\1\\0 \end{bmatrix}$ and $\boldsymbol{\nu} = \begin{bmatrix} 0\\0\\1 \end{bmatrix}$. (26)

If we seek anisotropy that generates no ISO component for shear faulting with any orientation, we obtain the following five conditions for the 15 elastic parameters of triclinic anisotropy (see Appendix A):

$$C_{11} + C_{12} - C_{23} - C_{33} = 0, (27)$$

$$C_{11} + C_{13} - C_{22} - C_{23} = 0, (28)$$

$$C_{16} + C_{26} + C_{36} = 0, (29)$$

$$C_{15} + C_{25} + C_{35} = 0, (30)$$

$$C_{14} + C_{24} + C_{34} = 0. ag{31}$$

The five conditions (27)–(31) for triclinic anisotropy reduce to three conditions (27)–(29) for monoclinic anisotropy, to two conditions (27)–(28) for orthorhombic anisotropy and only to condition (27) for trigonal anisotropy, tetragonal anisotropy or transverse isotropy (TI). Interestingly, any cubic anisotropy satisfies eqs (27)–(31), hence no shear faulting can generate the ISO component in this symmetry.

Because parameters C_{44} , C_{45} , C_{46} , C_{55} , C_{56} and C_{66} are missing in eqs (27)–(31), we conclude that the ISO component is not sensitive to them.

3 FAULT PLANE SOLUTIONS

We distinguish two different coordinate systems for seismic sources in anisotropic media. The first coordinate system is defined by the eigenvectors of source tensor **D**, which is related to the geometry of faulting. The other coordinate system is defined by the eigenvectors of moment tensor **M**, which is related to stresses generated by faulting. Both systems coincide in isotropy, but can differ in anisotropy. The deviation between them depends on the symmetry and strength of anisotropy and on geometry of faulting.

3.1 Eigenvectors of source tensor D

Source tensor **D**,

$$\mathbf{D} = \frac{uS}{2} (\mathbf{n}\boldsymbol{\nu} + \boldsymbol{\nu}\mathbf{n})$$

= $\frac{uS}{2} \begin{bmatrix} 2n_1v_1 & n_1v_2 + n_2v_1 & n_1v_3 + n_3v_1 \\ n_1v_2 + n_2v_1 & 2n_2v_2 & n_2v_3 + n_3v_2 \\ n_1v_3 + n_3v_1 & n_2v_3 + n_3v_2 & 2n_3v_3 \end{bmatrix}$, (32)

has the following diagonal form (see Appendix B):

$$\mathbf{D}^{\text{diag}} = \frac{uS}{2} \begin{bmatrix} \mathbf{n} \cdot \boldsymbol{\nu} + 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{n} \cdot \boldsymbol{\nu} - 1 \end{bmatrix},$$
(33)

where $\mathbf{n} \cdot \boldsymbol{\nu}$ is the scalar product of two unit vectors \mathbf{n} and $\boldsymbol{\nu}$,

$$\mathbf{n} \cdot \boldsymbol{\nu} = n_1 \nu_1 + n_2 \nu_2 + n_3 \nu_3. \tag{34}$$

The determinant of **D** is zero. The trace of **D** is

$$\operatorname{Trace}(\mathbf{D}) = D_{kk} = uS(\mathbf{n} \cdot \boldsymbol{\nu}). \tag{35}$$

The maximum eigenvalue $D_1 = \frac{uS}{2}(\mathbf{n} \cdot \boldsymbol{\nu} + 1)$ is positive or zero; the minimum eigenvalue $D_3 = \frac{uS}{2}(\mathbf{n} \cdot \boldsymbol{\nu} - 1)$ is negative or zero. The eigenvectors \mathbf{e}_1 , \mathbf{e}_2 and \mathbf{e}_3 of \mathbf{D} are (see Appendix B)

$$\mathbf{e}_1 = \frac{\mathbf{n} + \nu}{|\mathbf{n} + \nu|}, \quad \mathbf{e}_2 = \mathbf{n} \otimes \nu, \quad \mathbf{e}_3 = \frac{\mathbf{n} - \nu}{|\mathbf{n} - \nu|},$$
 (36)

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where symbol \otimes denotes the vector product. Eqs (33) and (36) apply to tensile sources (**n** and ν have arbitrary orientations) as well as to shear sources (**n** and ν are perpendicular). For shear sources, **D**^{diag} simplifies to the following form:

$$\mathbf{D}^{\text{diag}} = \frac{uS}{2} \begin{bmatrix} +1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1 \end{bmatrix}.$$
 (37)

3.2 Eigenvectors of moment tensor M

The eigenvectors of moment tensor **M** define the coordinate system, in which **M** diagonalizes:

$$\mathbf{M}^{\text{diag}} = \begin{bmatrix} M_1 & 0 & 0\\ 0 & M_2 & 0\\ 0 & 0 & M_3 \end{bmatrix}, \quad \text{where } M_1 \ge M_2 \ge M_3.$$
(38)

They are denoted as \mathbf{p} , \mathbf{t} and \mathbf{b} , and define the *P*, *T* and *B* axes. The *P*-axis corresponds to the minimum eigenvalue M_3 , the *T*-axis corresponds to the maximum eigenvalue M_1 and the *B*-axis corresponds to the intermediate eigenvalue M_2 . Physically, the *P*, *T* and *B* axes specify directions of the maximum compressional, maximum tensional and intermediate stresses generated at the source. These directions are generally different from the eigenvectors of source tensor \mathbf{D} . For isotropic media, the elastic parameters are specified as

$$c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$$
(39)

and eq. (4) yields

$$M_{ij} = \lambda D_{kk} \delta_{ij} + 2\mu D_{ij}, \tag{40}$$

$$\operatorname{Trace}(\mathbf{M}) = M_{kk} = (3\lambda + 2\mu)D_{kk},\tag{41}$$

where λ and μ are the Lamé coefficients. Inserting the diagonal source tensor (33) into eq. (40), we obtain

$$\mathbf{M} = uS \begin{bmatrix} (\lambda + \mu)\mathbf{n} \cdot \boldsymbol{\nu} + \mu & 0 & 0\\ 0 & \lambda \mathbf{n} \cdot \boldsymbol{\nu} & 0\\ & & (\lambda + \mu)\mathbf{n} \cdot \boldsymbol{\nu} - \mu \end{bmatrix}.$$
 (42)

This implies that tensor **M** has different eigenvalues than tensor **D**, but both tensors diagonalize in the same coordinate system. Therefore, the *P*, *T* and *B* axes can directly be inferred from fault normal **n** and slip direction ν :

$$\mathbf{b} = \mathbf{n} \otimes \boldsymbol{\nu}, \quad \mathbf{t} = \frac{\mathbf{n} + \boldsymbol{\nu}}{|\mathbf{n} + \boldsymbol{\nu}|}, \quad \mathbf{p} = \frac{\mathbf{n} - \boldsymbol{\nu}}{|\mathbf{n} - \boldsymbol{\nu}|}. \tag{43}$$

For shear sources, tensor **M** reads

$$\mathbf{M}^{\text{diag}} = \mu u S \begin{bmatrix} +1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$
(44)

well known as the DC moment tensor.

Table 1. Anisotropy models.

Model/rock	Туре	v^P	v^S	ρ	Sample/model identification	Reference	
		$({\rm km}~{\rm s}^{-1})$	$({\rm km}~{\rm s}^{-1})$	$(g cm^{-3})$			
				Theoretica	l models		
Dry cracks	TI	3.92	2.33	2.80	Model 3	Shearer & Chapman (1989)	
Water-filled cracks	TI	4.42	2.39	2.80	Model 1	Shearer & Chapman (1989)	
Periodic thin layers	TI	3.27	1.84	2.60	PTL2	Baptie et al. (1993, table 1)	
				Upper crus	stal rocks		
Sandstone	TI	4.57	2.61	2.46	Mesaverde (5481.3) immature sandstone	Thomsen (1986, table 1)	
Shale I	TI	4.25	2.37	2.50	Bazhenov shale (12.507 ft)	Vernik & Liu (1997, appendix A)	
Shale II	TI	3.33	1.74	2.42	Shale (5000 ft)	Jones & Wang (1981, table 2)	
Granite	ORT	5.29	3.13	2.64	Westerly granite	Lebedev et al. (2003, table 5b)	
Gneiss	TI	5.57	3.33	2.75	KTB (7.9–8.2 km)	Rabbel et al. (2004, table 1)	
Schist	TI	5.97	3.58	2.72	Haast schist, A-1 Godfrey <i>et al.</i> (2000, table 1 P = 100 MPa		
Phyllite	TI	6.17	3.71	2.74	Chugach phyllite, TA-23 $P = 100 \text{ MPa}$	Godfrey et al. (2000, table 1)	
Slate	TI	5.89	3.20	2.81	Vermont slate, VT-1 P = 100 MPa	Godfrey et al. (2000, table 1)	
				Lower crus	stal rocks		
Metapelite	ORT	7.59	4.25	3.06	Metapelite I, No. 3279	Weiss et al. (1999, table 1)	
Mafic granofels	ORT	7.00	3.89	2.94	Mafic granofels II, No. 3366	Weiss et al. (1999, table 1)	
Bt-plg gneiss	ORT	6.59	3.59	2.81	Bt-plg gneiss, No. 3270	Weiss et al. (1999, table 1)	
Amphibolite	TI	6.66	3.72	3.13	Green hornblende amphibolite, SB-1	Takanashi et al. (2001, table 4)	
Granulite	ORT	6.88	3.78	2.87	Granulite, No. 3261	Weiss et al. (1999, table 1)	
			Ma	ntle rocks and s	subduction zones		
Olivine aggregate I	ORT	7.99	4.56	3.30	Subduction zone average	Ben Ismail & Mainprice (1998, table 2)	
Olivine aggregate II	ORT	7.94	4.53	3.33	Total database average	Ben Ismail & Mainprice (1998, table 2)	
Xenolith I	ORT	7.95	4.62	3.23	KL, West Kettle river	Saruwatari et al. (2001, table 2)	
Xenolith II	ORT	7.93	4.49	3.31	Torre Alfina xenoliths	Pera et al. (2003, table 3)	
Tonga deep zone	ORT	10.37	5.80	3.92	Average slab model for depths of 500–700 km	Vavryčuk (2004, table 5)	

TI is transverse isotropy; ORT is orthorhombic anisotropy; v^P , v^S are average P and S velocities.

Table 2. Elastic parameters.

Model/rock	Туре	C_{11}	C ₂₂	C_{33}	C_{44}	C_{55}	C_{66}	C_{12}	C_{13}	C_{23}
				Theoretic	al models					
Dry cracks	TI	53.51	53.51	33.35	14.28	14.28	17.86	17.79	12.32	12.32
Water-filled cracks	TI	56.62	56.62	56.11	14.28	14.28	17.86	20.90	20.75	20.75
Periodic thin layers	TI	32.27	32.27	24.84	7.95	7.95	10.18	11.91	9.51	9.51
				Upper cru	istal rocks					
Sandstone	TI	55.00	55.00	46.53	16.26	16.26	19.68	15.65	20.89	20.89
Shale I	TI	58.81	58.81	27.23	13.23	13.23	23.54	11.73	23.64	23.64
Shale II	TI	34.30	34.30	22.70	5.40	5.40	10.60	13.10	10.70	10.70
Granite	ORT	72.27	69.00	75.06	27.31	26.46	24.92	21.25	23.84	22.13
Gneiss	TI	102.7	102.7	71.8	26.9	26.9	38.8	25.1	25.2	25.2
Schist	TI	113.8	113.8	88.5	29.9	29.9	41.8	30.2	23.0	23.0
Phyllite	TI	120.2	120.2	97.9	32.3	32.3	45.0	30.3	23.9	23.9
Slate	TI	123.3	123.3	80.5	21.1	21.2	46.0	31.3	40.4	40.4
				Lower cru	istal rocks					
Metapelite	ORT	167.0	188.8	178.0	58.6	51.5	52.7	61.3	62.8	66.4
Mafic granofels	ORT	141.6	157.9	140.4	47.0	41.8	42.7	57.5	53.9	53.6
Bt-plg gneiss	ORT	111.3	124.7	131.8	42.2	30.9	30.7	44.8	45.3	43.5
Amphibolite	TI	123.1	123.1	160.7	43.3	43.3	38.8	45.6	42.8	42.8
Granulite	ORT	142.1	131.3	135.7	40.9	41.1	40.3	53.9	56.3	51.0
			Ma	ntle rocks and	subduction z	ones				
Olivine aggregate I	ORT	192.1	237.1	208.8	72.6	63.3	68.5	69.9	72.4	73.5
Olivine aggregate II	ORT	195.5	236.9	205.9	71.4	63.0	69.8	71.1	72.1	72.4
Xenolith I	ORT	217.2	192.4	201.0	67.4	71.9	70.1	67.4	68.0	67.5
Xenolith II	ORT	187.8	240.5	206.5	70.2	60.6	66.9	70.7	73.8	73.0
Tonga deep zone	ORT	421.8	447.3	404.9	110.5	154.8	134.5	145.0	188.9	149.0

Elastic parameters C_{ij} are in 10⁹ kg m⁻¹ s⁻².

Dry cracks



Water-filled cracks



Figure 1. Phase velocities of *P* (upper plot) and *S* (lower plot) waves as a function of the deviation of the wave normal from the symmetry axis for the model of dry cracks.

Figure 2. Phase velocities of P (upper plot) and S (lower plot) waves as a function of the deviation of the wave normal from the symmetry axis for the model of water-filled cracks.



Figure 3. Directional variation of phase velocities (in km s⁻¹) for the model of olivine aggregate I. Equal-area projection is used.

3.3 Inversion for geometry of faulting

In order to determine fault normal **n** and slip direction ν from moment tensor **M** and from the matrix of elastic parameters **C**, we have to calculate vector **d** from eq. (6),

$$\mathbf{d} = \mathbf{C}^{-1}\mathbf{m},\tag{45}$$

and subsequently construct source tensor **D** using eq. (9). Diagonalizing **D**, we obtain eigenvalues D_1 and D_3 , and eigenvectors \mathbf{e}_1 , \mathbf{e}_2 and \mathbf{e}_3 . The angle δ between the fault normal and slip direction is determined from the trace of **D** (eq. 35):

$$\cos(\delta) = \frac{1}{uS} \operatorname{Trace}(\mathbf{D}) = \frac{D_1 + D_3}{D_1 - D_3}.$$
(46)

Vectors **n** and ν are determined from the eigenvectors and eigenvalues of **D**:

$$\mathbf{n} = \frac{1}{\sqrt{D_1 - D_3}} (\sqrt{|D_1|} \mathbf{e}_1 + \sqrt{|D_3|} \mathbf{e}_3), \tag{47}$$



Figure 4. Directional variation of phase velocities (in km s^{-1}) for the model of the Tonga deep zone. Equal-area projection is used.

$$\nu = \frac{1}{\sqrt{D_1 - D_3}} (\sqrt{|D_1|} \mathbf{e}_1 - \sqrt{|D_3|} \mathbf{e}_3).$$
(48)

For shear sources $(\mathbf{n} \perp \boldsymbol{\nu})$, Trace $(\mathbf{D}) = 0$, $D_1 = \frac{uS}{2}$ and $D_3 = -\frac{uS}{2}$, hence

$$\mathbf{n} = \frac{1}{\sqrt{2}} (\mathbf{e}_1 + \mathbf{e}_3),\tag{49}$$

$$\boldsymbol{\nu} = \frac{1}{\sqrt{2}} (\mathbf{e}_1 - \mathbf{e}_3). \tag{50}$$

It follows from the symmetry in the positions of fault normal **n** and slip direction ν in formulae (5) and (8) that the solution for **n** and ν is ambiguous, and the plus and minus signs in eqs (47)–(48) and in (49)–(50) can be interchanged.

4 NUMERICAL MODELLING

In this section, the properties of focal mechanisms and the calculation of fault plane solutions in anisotropic media will be illustrated numerically.

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Figure 5. The percentages of the non-DC components generated by shear faulting in the dry cracks model with an inclined symmetry axis. Geometry of faulting is fixed: $\mathbf{n} = (0, 0, 1)^T$, $\boldsymbol{\nu} = (1, 0, 0)^T$. Points inside the circle correspond to transverse isotropy (TI) with a varied orientation of the symmetry axis. The plus sign marks the TI with the vertical symmetry axis; the points along the circle correspond to TI with horizontal symmetry axes. The colour indicates the value of the non-DC component. Equal-area projection is used.

4.1 Anisotropy models

We consider 21 models of anisotropic media (see Table 1). First, we consider theoretical models of anisotropy inferred for cracked and layered media. The cracked media contain aligned dry (empty) or water-filled cracks and the effective anisotropy is calculated using the Hudson (1981) theory. The layered medium is composed of periodic thin layers and the effective anisotropy is calculated by the Backus (1962) averaging. The media display transverse isotropy and should characterize rocks in the uppermost crust of the Earth. Secondly, we consider anisotropy models of transverse isotropy (TI) or of orthorhombic symmetry (ORT) inferred from laboratory measurements of velocities on various rock samples. The rocks originate from the upper and lower crust, and from the upper mantle and subduction zones. The samples include a variety of sedimentary, metamorphic and crustal igneous rocks, as well as ultramafic mantle rocks, with a wide range of compositions and microstructures, and the velocities were measured under varied stress conditions. Finally, we consider an ORT anisotropy model inferred from in situ observations. The anisotropy model was determined for the deep part of the Tonga subduction zone using an inversion of the non-DC components of moment tensors of deep-focus earthquakes (Vavryčuk 2004). Obviously, the presented models do not cover all possible variations of anisotropy that might occur at focal areas, but still they can give an insight into the problem of how significantly



Figure 6. The percentages of the non-DC components generated by shear faulting in the water-filled cracks model with an inclined symmetry axis. Equal-area projection is used. The plus sign marks the vertical axis. For details, see the caption of Fig. 5.

focal mechanisms and seismic moment tensors can be affected by anisotropy.

Table 1 presents the type of anisotropy, average P and S velocities, the density, the identification code and the reference for each rock sample or anisotropy model. Table 2 presents the elastic parameters of the rock samples or models. Several authors (Ben Ismail & Mainprice 1998; Weiss *et al.* 1999; Saruwatari *et al.* 2001; Pera *et al.* 2003) published a complete elasticity tensor, which describes triclinic anisotropy and includes 21 elastic parameters. Because all these samples were very close to orthorhombic anisotropy, Table 2 presents only the orthorhombic subset of the elastic parameters. Thomsen (1986), Vernik & Liu (1997) and Takanashi *et al.* (2001) published anisotropy of the rock samples using weak anisotropy parameters. For these samples, the elastic parameters were calculated using Thomsen's formulae (Thomsen 1986).

A directional variation of phase velocities is exemplified for two TI and two ORT models (see Figs 1–4). Figs 1 and 2 show the phase velocities for the dry and water-filled crack models (see Shearer & Chapman 1989). The anisotropy strength of the *P*, *SV* and *SH* waves is 23.5, 1.3 and 11.2 per cent for dry cracks, and 3.5, 11.0 and 11.2 per cent for the water-filled cracks. Fig. 3 shows the phase velocities for the model of the olivine aggregate I (see Ben Ismail & Mainprice 1998), which displays anisotropy of 9.6, 3.0 and 5.6 per cent for the *P*, *S*1 and *S*2 waves, respectively. Fig. 4 shows the *P*, *S*1 and *S*2 phase velocities for the model of the Tonga deep zone with anisotropy of 7.3, 13.4 and 12.6 per cent, respectively (see Vavryčuk 2004).



Figure 7. Histograms of the CLVD components produced by shear faulting in olivine aggregate I (a) and in the Tonga deep zone (b).

4.2 Non-DC components

The non-DC components generated by shear faulting in anisotropic media depend on the type and strength of anisotropy and on the orientation of faulting. For TI, the behaviour of the non-DC components is illustrated on shear faulting in cracked media (see Figs 5 and 6). The geometry of faulting is fixed: the fault normal lies along the z-axis and the slip along the x-axis, $\mathbf{n} = (0, 0, 1)^T$ and $\boldsymbol{\nu} = (1, 0, 0, 1)^T$ $(0, 0)^T$. Such a source generates no ISO and no CLVD for TI with a vertical symmetry axis. However, if the symmetry axis is inclined, the ISO and CLVD become non-zero. The values of ISO and CLVD are shown as a function of the direction of the symmetry axis in Fig. 5 (dry cracks) and Fig. 6 (water-filled cracks). For the model of dry cracks, the percentages of the ISO and CLVD are in the intervals (-20.7, 20.7) and (-16.1, 16.1), respectively. For the model of water-filled cracks, the percentages of the ISO and CLVD are in the intervals (-0.6, 0.6) and (-19.9, 19.9), respectively. Hence, the shear source produces remarkable non-DC components in both anisotropy models. For dry cracks, both the ISO and CLVD are high. On the contrary, water-filled cracks produce high CLVD, but almost zero ISO. Despite the different percentages of the ISO in both models, their directional variation is similar. Interestingly, the directional variations of CLVD are quite different for both models, the variation for water-filled cracks being more complicated than for dry cracks.

In order to estimate, how large non-DC components can be produced by shear faulting in the anisotropy models presented in Table 1, we used the following procedure: we generated a set of 10 000 randomly oriented mechanisms defined by angles dip, strike and rake. For this set, we calculated moment tensors in each anisotropy



Figure 8. Histograms of the ISO component produced by shear faulting in olivine aggregate I (a) and in the Tonga deep zone (b).

model using eq. (3) and decomposed it into the DC, CLVD and ISO using eqs (15)–(17). From the obtained sets of the DC and non-DC values, we calculated the maximum absolute value of the CLVD and ISO, and the minimum value of the DC. Figs 7 and 8 demonstrate the behaviour of the CLVD and ISO in the olivine aggregate I and in the Tonga deep zone. Table 3 summarizes the extreme values of the DC, CLVD and ISO in all anisotropy models. The table shows that, except for two models (dry cracks and granulite), the CLVD is more significant than the ISO. Typically, the CLVD attains values from 15 to 30 per cent and ISO from 5 to 15 per cent. The highest value of CLVD is 83 per cent in shale I and the lowest value is 2 per cent in granulite. The highest value of ISO is almost 21 per cent for dry cracks and 0.6 per cent for water-filled cracks.

4.3 Correction for a true fault normal and slip direction

So far, fault plane solutions are calculated in the seismological practice under the assumption of an isotropic focal area. If the focal area is anisotropic, the procedure yields distorted results. The errors owing to neglect of anisotropy are illustrated in Fig. 9 (dry cracks) and in Fig. 10 (water-filled cracks). The upper/lower plots in the figures show the deviation between the true and approximate fault normals/slip directions. The approximate values were calculated from the eigenvectors of the moment tensor using the following standard formulae:

$$\mathbf{n}^{\text{approx}} = \frac{1}{\sqrt{2}}(\mathbf{p} + \mathbf{t}),\tag{51}$$

$$\nu^{\text{approx}} = \frac{1}{\sqrt{2}} (\mathbf{p} - \mathbf{t}), \tag{52}$$

Table 3. Average velocities, anisotropy strength and non-double-couple (non-DC) components produced by shear faulting.

e	· · · ·	1.5	0		, I		5	e		
Model/rock	Type	v^P	v^S	a^P	a^{SV}/a^{S1}	a^{SH}/a^{S2}	CLVD ^{MAX}	ISO ^{MAX}	DC ^{MIN}	δ^{MAX}
		$(\mathrm{km}\ \mathrm{s}^{-1})$	$(\mathrm{km}~\mathrm{s}^{-1})$	(per cent)	(per cent)	(per cent)	(per cent)	(per cent)	(per cent)	(°)
				Theor	retical models					
Dry cracks	TI	3.92	2.33	23.5	1.3	11.2	16.1	20.7	64.3	6.4
Water filled cracks	TI	4.42	2.39	3.5	11.0	11.2	19.9	0.6	79.8	6.4
Periodic thin layers	TI	3.27	1.84	13.1	8.1	12.3	18.7	14.4	72.0	7.1
				Uppe	r crustal rocks					
Sandstone	TI	4.57	2.61	8.4	4.7	9.5	37.1	3.2	59.8	6.7
Shale I	TI	4.25	2.37	38.0	26.1	28.6	83.2	18.6	2.0	62.1
Shale II	TI	3.33	1.74	20.8	22.4	33.4	40.9	19.8	46.0	19.0
Granite	ORT	5.29	3.13	4.5	3.6	3.5	9.8	5.4	89.4	2.6
Gneiss	TI	5.57	3.33	17.8	5.4	18.3	27.5	13.2	60.0	10.4
Schist	TI	5.97	3.58	13.1	12.5	16.7	25.2	11.9	67.6	9.5
Phyllite	TI	6.17	3.71	11.4	13.2	16.5	25.5	9.9	68.7	9.5
Slate	TI	5.89	3.20	21.2	16.1	38.5	50.4	13.6	37.1	21.8
				Lowe	r crustal rocks					
Metapelite	ORT	7.59	4.25	6.2	5.3	4.6	12.9	6.6	82.3	3.7
Mafic granofels	ORT	7.00	3.89	6.3	5.4	4.0	12.6	6.7	81.6	3.5
Bt-plg gneiss	ORT	6.59	3.59	8.9	15.7	9.9	25.2	7.3	68.9	9.1
Amphibolite	TI	6.66	3.72	13.3	5.8	5.5	24.4	9.8	65.7	5.2
Granulite	ORT	6.88	3.78	4.0	0.5	0.9	2.2	6.1	93.7	0.6
				Mantle rocks	and subduction	n zones				
Olivine aggregate I	ORT	7.99	4.56	10.5	4.1	5.2	17.1	9.2	73.8	3.9
Olivine aggregate II	ORT	7.94	4.53	9.6	3.0	5.6	16.8	8.4	75.2	3.7
Xenolith I	ORT	7.95	4.62	6.1	2.2	4.1	10.6	5.6	83.8	2.7
Xenolith II	ORT	7.93	4.49	12.4	5.5	6.1	21.3	10.2	68.6	4.8
Tonga deep zone	ORT	10.37	5.80	7.3	13.4	12.6	28.7	1.8	71.2	9.6

 $\overline{a^P, a^{SV}/a^{S1}, a^{SH}/a^{S2}}$ denote the anisotropy strength for *P*, *SV* and *SH* waves in the case of TI anisotropy and the anisotropy strength for *P*, *S*1 and *S*2 waves in the case of ORT anisotropy. The anisotropy strength is defined as $a = 200 (v^{MAX} - v^{MIN})/(v^{MAX} + v^{MIN})$, where v^{MAX} and v^{MIN} are the maximum and minimum phase velocities of the respective wave. CLVD^{MAX}, ISO^{MAX}, DC^{MIN} and δ^{MAX} are the maximum absolute values of the compensated linear vector dipole (CLVD) and isotropic (ISO) components, the minimum value of the double couple (DC) and the maximum deviation of the approximate fault normal and slip direction from true values observed in the specified anisotropy, respectively.

where **p** and **t** are the unit eigenvectors of the moment tensor corresponding to the *P* and *T* axes. Because of the ambiguity of the solutions (51)–(52) for the fault normal and slip, we selected the pair of $\mathbf{n}^{\text{approx}}$ and $\boldsymbol{\nu}^{\text{approx}}$ that approximated the true vectors better.

Figs 9 and 10 show that the maximum deviation between the true and approximate fault normals and slips attains coincidently a value of 6.4° . This indicates that the deviation is not sensitive to the strength of the *P*-wave anisotropy, which is different in both models, but rather to the strength of the *S*-wave anisotropy. The maximum deviation of 6.4° , corresponding to the *S*-wave anisotropy of 11.2 per cent, is not very high, but it can still introduce a non-negligible bias in carefully determined focal mechanisms. Similarly as for the CLVD, the directional variation of the deviations is more complicated for water-filled cracks than for dry cracks.

The maximum possible deviations of true fault normals or true slip directions from approximate values calculated using an isotropic procedure are summarized in Table 3. The deviations were calculated similarly as the values of the DC, CLVD and ISO in Section 4.2 from a set of 10 000 randomly oriented focal mechanisms calculated in each anisotropy model. The table indicates that the maximum deviation is typically less than 10° . The exceptions are shale I, shale II, slate and gneiss with deviations of 62.1°, 19.0°, 21.8° and 10.4°, respectively. All models with the anomalous deviations can be considered as strongly anisotropic.

5 CONCLUSIONS

(i) Shear faulting on planar faults in anisotropic media can produce non-DC mechanisms. The non-DC mechanisms can comprise both the CLVD and ISO components. The amount of the CLVD and ISO depends on strength and symmetry of anisotropy and on the orientation of faulting.

(ii) Under specific conditions imposed on anisotropy, shear faulting with an arbitrary orientation generates no ISO components. These conditions are satisfied, for example, in any cubic anisotropy. The ISO components produced by shear faulting in triclinic anisotropy are not sensitive to parameters C_{44} , C_{45} , C_{46} , C_{55} , C_{56} and C_{66} .

(iii) In some specific cases, shear faulting in anisotropy can also produce pure DC mechanisms similarly as in isotropy. This happens, for example, when faulting is situated in the horizontal plane of monoclinic anisotropy or in any symmetry plane of orthorhombic anisotropy or of higher symmetries. However, the scalar seismic moment can vary with respect to the orientation of the slip in a symmetry plane. The double couple can deviate from the plane defined by the fault normal and slip direction.

(iv) The fault plane solutions in anisotropy display the same ambiguity in identifying a fault normal and slip direction as in isotropy. The moment tensor corresponding to shear faulting in anisotropy has eigenvectors (P, T and B axes), which can deviate from those





Figure 9. The deviation between the true and approximate fault normals (upper plot), and between the true and approximate slip directions (lower plot) for the dry cracks model with an inclined symmetry axis. The centre of the circle corresponds to transverse isotropy (TI) with the vertical symmetry axis; the points along the circle correspond to TI with horizontal symmetry axes. The colour indicates the angular deviation. Equal-area projection is used. The deviation is in degrees.

in isotropy. If anisotropy is neglected at a focal area, the fault plane and slip calculated under the isotropic assumption can deviate from true ones.

(v) Shear faulting in anisotropic rocks present in the Earth crust, the Earth mantle or in the subduction zones can produce mechanisms with significant non-DC components. The CLVD can typically attain values up to 30 per cent and the ISO up to 15 per cent. The fault plane solutions calculated under the assumption of isotropy typically deviate from the true solutions by an angle of 5° to 10° . However, for strongly anisotropic rocks such as some shales or schists, the CLVD and ISO can be much higher, and the isotropic procedure for determining the fault plane solution can completely fail.

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Figure 10. The deviation between the true and approximate fault normals (upper plot), and between the true and approximate slip directions (lower plot) for the water-filled cracks model with an inclined symmetry axis. For details, see the caption of Fig. 9.

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APPENDIX A: ANISOTROPY GENERATING NO ISO COMPONENTS FOR SHEAR FAULTING WITH AN ARBITRARY ORIENTATION

Assume shear faulting in triclinic anisotropy with fault normal $\mathbf{n} = (0, 0, 1)^T$ and slip direction $\boldsymbol{\nu} = (1, 0, 0)^T$. According to eq. (18), the moment tensor of the source is expressed as follows:

$$\mathbf{M} = uS \begin{bmatrix} C_{15}^* & C_{56}^* & C_{55}^* \\ & C_{25}^* & C_{45}^* \\ & & C_{35}^* \end{bmatrix}.$$
 (A1)

The zero ISO component is equivalent to the condition of zero trace of M:

Trace(**M**) =
$$uS(C_{15}^* + C_{25}^* + C_{35}^*) = 0$$
,

(A2)

where the matrix of elastic parameters C^* refers to the coordinate system, in which two axes (x_1 and x_3) are formed by vectors **n** and ν . If the orientation of the coordinate system is arbitrary with respect to the orientation of shear faulting, elasticity tensor c_{ijkl} must be rotated (see Musgrave 1970, eq. 3.12.2a). Using the rotated coordinate system

$$\mathbf{e}_{1} = \begin{bmatrix} -\sin\psi\sin\varphi\cos\theta + \cos\psi\cos\varphi \\ -\sin\psi\cos\varphi\cos\theta - \cos\psi\sin\varphi \\ \sin\psi\sin\theta \end{bmatrix},\tag{A3}$$

$$\mathbf{e}_{2} = \begin{bmatrix} \sin\psi\cos\varphi + \cos\psi\sin\varphi\cos\theta \\ -\sin\psi\sin\varphi + \cos\psi\cos\varphi\cos\theta \\ -\cos\psi\sin\theta \end{bmatrix},\tag{A4}$$

$$\mathbf{e}_{3} = \begin{bmatrix} \sin\varphi\sin\theta\\ \cos\varphi\sin\theta\\ \cos\theta \end{bmatrix},\tag{A5}$$

where ψ , φ and θ are angles defining the rotation, the trace of **M** reads

 $\operatorname{Trace}(\mathbf{M}) = uS(C_{11} + C_{12} - C_{23} - C_{33})\sin\varphi\sin\theta\cos\theta$

 $-uS(C_{11}+C_{13}-C_{22}-C_{23})\cos\psi\sin\theta(\sin\psi\cos\varphi+\cos\psi\sin\varphi\cos\theta)$

$$-uS(C_{14} + C_{24} + C_{34})(\sin\psi\cos\varphi\cos\theta + \cos\psi\sin\varphi\cos2\theta)$$

 $-uS(C_{15}+C_{25}+C_{35})(\cos\psi\cos\varphi\cos\theta-\sin\psi\sin\varphi\cos2\theta)$

$$-uS(C_{16}+C_{26}+C_{36})\bigg(\frac{1}{2}\sin 2\psi\sin\varphi\sin 2\theta-\cos 2\psi\cos\varphi\sin\theta\bigg).$$
(A6)

The condition of zero trace of **M** for any value of angles ψ , φ and θ yields

$$C_{11} + C_{12} - C_{23} - C_{33} = 0, (A7)$$

$$C_{11} + C_{13} - C_{22} - C_{23} = 0, (A8)$$

$$C_{14} + C_{24} + C_{34} = 0, (A9)$$

$$C_{15} + C_{25} + C_{35} = 0, (A10)$$

$$C_{16} + C_{26} + C_{36} = 0. (A11)$$

Eqs (A7)-(A11) represent a system of five equations for 15 elastic parameters of triclinic anisotropy.

APPENDIX B: EIGENVALUES AND EIGENVECTORS OF SOURCE TENSOR D

A spectral decomposition of tensor **D** reads

$\mathbf{D} = D_1 \mathbf{e}_1 \mathbf{e}_1 + D_2 \mathbf{e}_2 \mathbf{e}_2 + D_3 \mathbf{e}_3 \mathbf{e}_3,$	(B1)

where D_1 , D_2 and D_3 are the eigenvalues, and \mathbf{e}_1 , \mathbf{e}_2 and \mathbf{e}_3 are the eigenvectors of **D**. The eigenvectors are of a unit length and mutually perpendicular. Because source tensor **D** is formed by a dyad of vectors **n** and $\boldsymbol{\nu}$,

$$\mathbf{D} = \frac{uS}{2} (\mathbf{n}\boldsymbol{\nu} + \boldsymbol{\nu}\mathbf{n}),\tag{B2}$$

it is easy to show that one of egenvectors of **D** is perpendicular to **n** and ν , and the corresponding eigenvalue is zero:

$$\mathbf{e}_2 = \mathbf{u} \otimes \boldsymbol{\nu}, \quad D_2 = \mathbf{0}, \tag{B3}$$

where symbol \otimes denotes the vector product. Because $\mathbf{e}_1 \perp \mathbf{e}_2 \perp \mathbf{e}_3$, vectors \mathbf{e}_1 and \mathbf{e}_3 must lie in the plane defined by vectors \mathbf{n} and $\boldsymbol{\nu}$. It follows from the following dyadic products,

$$(\nu + \mathbf{n})(\nu + \mathbf{n}) = \nu\nu + \mathbf{n}\mathbf{n} + \nu\mathbf{n} + \mathbf{n}\nu, \tag{B4}$$

$$(\nu - \mathbf{n})(\nu - \mathbf{n}) = \nu\nu + \mathbf{n}\mathbf{n} - \nu\mathbf{n} - \mathbf{n}\nu,$$
(B5)

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that tensor **D** can be obtained by their combinations, and thus the eigenvectors \mathbf{e}_1 and \mathbf{e}_3 can be expressed as follows:

$$\mathbf{e}_1 = \frac{\boldsymbol{\nu} + \mathbf{n}}{|\boldsymbol{\nu} + \mathbf{n}|}, \quad \mathbf{e}_3 = \frac{\boldsymbol{\nu} - \mathbf{n}}{|\boldsymbol{\nu} - \mathbf{n}|}.$$
 (B6)

Inserting eq. (B6) into eq. (B1), we get

$$\mathbf{D} = D_1 \frac{(\nu + \mathbf{n})(\nu + \mathbf{n})}{(\nu + \mathbf{n}) \cdot (\nu + \mathbf{n})} + D_3 \frac{(\nu - \mathbf{n})(\nu - \mathbf{n})}{(\nu - \mathbf{n}) \cdot (\nu - \mathbf{n})}$$

$$= \left[\frac{D_1}{(\nu + \mathbf{n}) \cdot (\nu + \mathbf{n})} + \frac{D_3}{(\nu - \mathbf{n}) \cdot (\nu - \mathbf{n})}\right] (\nu \nu + \mathbf{n}\mathbf{n}) + \left[\frac{D_1}{(\nu + \mathbf{n}) \cdot (\nu + \mathbf{n})} - \frac{D_3}{(\nu - \mathbf{n}) \cdot (\nu - \mathbf{n})}\right] (\nu \mathbf{n} + \mathbf{n}\nu), \tag{B7}$$

where the dot between two vectors means its scalar product. Comparing with eq. (B2), we obtain the equations for D_1 and D_3 :

$$\left[\frac{D_1}{(\boldsymbol{\nu}+\mathbf{n})\cdot(\boldsymbol{\nu}+\mathbf{n})} + \frac{D_3}{(\boldsymbol{\nu}-\mathbf{n})\cdot(\boldsymbol{\nu}-\mathbf{n})}\right](\boldsymbol{\nu}\boldsymbol{\nu}+\mathbf{n}\mathbf{n}) = 0,$$
(B8)

$$\left[\frac{D_1}{(\boldsymbol{\nu}+\mathbf{n})\cdot(\boldsymbol{\nu}+\mathbf{n})} - \frac{D_3}{(\boldsymbol{\nu}-\mathbf{n})\cdot(\boldsymbol{\nu}-\mathbf{n})}\right](\boldsymbol{\nu}\mathbf{n}+\mathbf{n}\boldsymbol{\nu}) = \frac{uS}{2}.$$
(B9)

From eqs (B8)-(B9), we write

$$D_1 = \frac{uS}{4}(\boldsymbol{\nu} + \mathbf{n}) \cdot (\boldsymbol{\nu} + \mathbf{n}), \tag{B10}$$

$$D_3 = -\frac{uS}{4}(\boldsymbol{\nu} - \mathbf{n}) \cdot (\boldsymbol{\nu} - \mathbf{n}). \tag{B11}$$

Taking into account that $\nu \cdot \nu = \mathbf{n} \cdot \mathbf{n} = 1$, we finally get for eigenvalues of **D**

$$D_1 = \frac{uS}{2}(\boldsymbol{\nu} \cdot \mathbf{n} + 1), \quad D_2 = 0, \quad D_3 = \frac{uS}{2}(\boldsymbol{\nu} \cdot \mathbf{n} - 1),$$
 (B12)

the eigenvectors of **D** being defined in eqs (B3) and (B6).