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Is the seismic moment tensor ambiguous at a material interface?

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SUMMARY

The moment tensors are unique and describe the body force equivalents of rupture processes in any medium including faulting at a material interface, defined as the contact of two media with non-zero velocity or density contrasts. From a practical point of view, however, the moment tensor inversion of sources near or at a material interface is more involved than if the medium is smooth in the source area. First, the moment tensors of sources characterized by the same displacement discontinuity display jumps when the source crosses the interface. Consequently, the moment tensors become sensitive to the source location. If the source lies near the interface, the location into an incorrect half-space can introduce errors in the moment tensor. Second, if the source lies at the material interface, some of the spatial derivatives of the Green's function are, in general, discontinuous and the radiated wave field must be calculated using a generalized representation theorem. Third, the moment tensors are functions of averaged elastic parameters known from effective medium theory. The theory implies that shear faulting at a material interface in isotropic media is represented by the standard double-couple moment tensor. The scalar seismic moment is calculated as a product of the displacement discontinuity across the fault, the fault size and the effective rigidity at the fault. The effective rigidity is the harmonic mean of rigidities at the individual sides of the fault.

Key words: Earthquake dynamics; Earthquake source observations; Body waves; Seismic anisotropy; Dynamics and mechanics of faulting.

1 INTRODUCTION

The moment tensor was introduced as a mathematical representation of forces associated with an earthquake rupture (Burridge & Knopoff 1964; Backus & Mulcahy 1976a,b). The forces described by the moment tensor, however, are not the actual forces acting at the source. Since the rheological properties of the material at the fault are usually strongly non-linear and known vaguely, a proper stress–strain relation cannot be established, and the actual forces in such a complex environment cannot be accurately retrieved. Nevertheless, the non-linear behaviour of the material is mainly limited just to a fault gouge and the surrounding medium can be described well by elastic or viscoelastic parameters. For this reason, the moment tensor ignores the forces associated with the non-linear rheology at the fault and with rupturing itself. The fault is assumed to be an artificial internal surface inside the elastic medium along which the body forces are distributed producing the waves and static deformations in the medium outside the fault. This description proved to be useful and became widely used in seismological practice for quantifying seismic sources in the point source approximation. The moment tensor is now a standard quantity evaluated for earthquakes on all scales from acoustic emissions to large devastating earthquakes (Dziewonski *et al.* 1981; Sipkin 1982; Fukuyama *et al.* 2001; Pondrelli *et al.* 2002). The moment tensors are used to quantify the moment magnitude, double-couple as well as non-double-couple focal mechanisms (Frohlich 1994; Lay & Wallace 1995; Julian *et al.* 1998; Miller *et al.* 1998; Vavryčuk 2001, 2002, 2011; Kanamori & Brodsky 2004), and help understand physical processes at the earthquake source.

Even though the moment tensor is a fundamental quantity describing the earthquakes in the point-source approximation, some authors question the usefulness and unambiguity of the moment tensor. The confusions mostly arise for earthquakes at faults forming a contact of two media with non-zero velocity or density contrasts (Woodhouse 1981; Heaton & Heaton 1989a,b; Ben-Zion 2001; Ampuero & Dahlen 2005). Such earthquakes have been studied theoretically (Ben-Zion 1989, 1990; Ben-Zion & Andrews 1998; Shi & Ben-Zion 2009) as well as observed in various tectonic settings (Le Pichon *et al.* 2005; Houlie & Romanowicz 2011; Ozakin *et al.* 2012). The rupturing along the faults at a material discontinuity interface might also have some interesting consequences for the migration of aftershock activity (Rubin 2002; Rubin & Ampuero 2007; Zaliapin & Ben-Zion 2011).

The family of earthquakes at faults forming a material interface cannot be described using the standard theory of moment tensors. For example, theory predicts the scalar seismic moment

M_0 of shear faulting in an isotropic medium in the following form:

$$M_0 = \mu\,[u]\,S, \tag{1}$$

where μ is the rigidity of the medium, $[u]$ is the average slip (displacement discontinuity) on the fault and S is the fault size. If the rigidity of the medium is different at both sides of the fault, it is not obvious, how to evaluate the scalar moment. To overcome this difficulty, Wu & Chen (2003) proposed to substitute rigidity μ in eq. (1) with effective rigidity μ^*:

$$M_0 = \mu^*\,[u]\,S \tag{2}$$

expressed as

$$\mu^* = 2\frac{\mu^+\mu^-}{\mu^+ + \mu^-}, \tag{3}$$

where μ^+ and $\mu^-$are values of the rigidity at the individual sides of the fault. However, Ampuero & Dahlen (2005) argue that eqs (2) and (3) cannot be viewed as the only possible prescription of the scalar moment. They argue that theory is inherently ambiguous and does not constrain the scalar moment uniquely. They conclude that a more appropriate description of earthquake sources is obtained if source (potency) tensors are applied. These tensors avoid the problem of ambiguous rigidity μ because they are no longer related to forces but to deformations at the fault only.

In this paper, I reopen the debate about the ambiguity of the moment tensor for earthquakes occurring at a material interface. I show that correct application of theory leads to a unique moment tensor even for such a difficult case as that discussed earlier.

2 REPRESENTATION THEOREM FOR SOURCES IN A SMOOTH MEDIUM

2.1 Single body forces

The equation of motion in elastic anisotropic media reads

$$\rho\ddot{u}_i - \tau_{ij,j} = f_i, \tag{4}$$

where $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ is the displacement, $\rho = \rho(\mathbf{x})$ is the mass density, $\tau_{ij} = \tau_{ij}(\mathbf{x}, t)$ is the stress tensor, $\mathbf{f} = \mathbf{f}(\mathbf{x}, t)$ is the body force, $c_{ijkl} = c_{ijkl}(\mathbf{x})$ is the stiffness tensor, $\mathbf{x}$ is the position vector, and t is time. The stress tensor is expressed using Hooke's law as

$$\tau_{ij} = c_{ijkl}\varepsilon_{kl}, \tag{5}$$

where $\varepsilon_{kl} = \varepsilon_{kl}(\mathbf{x}, t)$ is the strain tensor

$$\varepsilon_{kl} = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right). \tag{6}$$

Eq. (4) is often solved using Green's function $G_{in} = G_{in}(\mathbf{x}, t; \boldsymbol{\xi}, \tau)$ defined as the solution of the following equation:

$$\rho\ddot{G}_{in} - \left(c_{ijkl}G_{kn,l}\right)_{,j} = \delta_{in}\delta(\mathbf{x} - \boldsymbol{\xi})\,\delta(t - \tau) \tag{7}$$

under appropriate boundary and initial conditions. Displacement $\mathbf{u}$ is expressed as

$$u_i(\mathbf{x}, t) = \int_{-\infty}^{\infty} \mathrm{d}\tau \iiint f_n(\boldsymbol{\xi}, \tau)G_{in}(\mathbf{x}, t; \boldsymbol{\xi}, \tau)\,dV(\boldsymbol{\xi}) \tag{8}$$

or simply

$$u_i = f_n * G_{in}, \tag{9}$$

where symbol '$*$' stands for the space–time convolution, and $\boldsymbol{\xi}$ and τ are the position vector and time at the source.

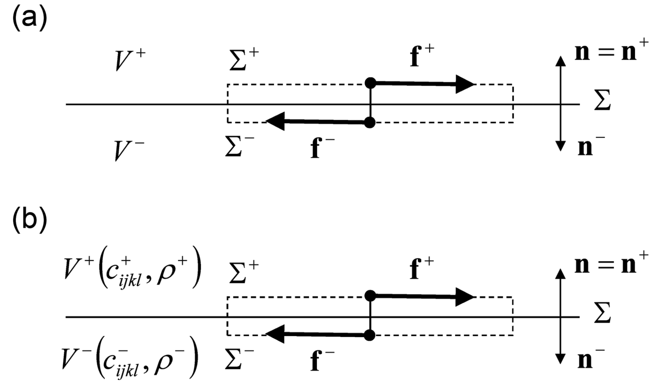

Figure 1. Force couple at a fault in a smooth medium (a) and at a material interface (b).

2.2 Dipole body forces

The body forces $\mathbf{f}(\boldsymbol{\xi}, \tau)$ associated with the earthquake source have some specific properties. First, the forces must satisfy a condition of the inner source. The condition requires the total force and the total torque at the source to be zero (Aki & Richards 2002, eqs 3.6 and 3.7):

$$\iiint_V \mathbf{f}(\boldsymbol{\xi}, \tau)\,dV(\boldsymbol{\xi}) = \mathbf{0},$$

$$\iiint_V (\boldsymbol{\xi} - \boldsymbol{\xi}_0) \times \mathbf{f}(\boldsymbol{\xi}, \tau)\,dV(\boldsymbol{\xi}) = \mathbf{0}, \tag{10}$$

for all τ and any fixed $\boldsymbol{\xi}_0$. Second, the forces are not distributed in a volume, but rather along fault Σ. And third, the source need not be necessarily represented by single forces but mostly by dipole forces.

For dipole forces, displacement $\mathbf{u}(\mathbf{x}, t)$ is obtained as the sum of displacements $\mathbf{u}^+(\mathbf{x}, t)$ and $\mathbf{u}^-(\mathbf{x}, t)$, produced by individual single forces $\mathbf{f}^+(\boldsymbol{\xi}^+, \tau)$ and $\mathbf{f}^-(\boldsymbol{\xi}^-, \tau)$, forming a force couple, $\mathbf{f}^+ = -\mathbf{f}^-$ (see Fig. 1a)

$$u_i(\mathbf{x}, t) = u_i^+(\mathbf{x}, t) + u_i^-(\mathbf{x}, t) = f_k^+ * G_{ik}^+ + f_k^- * G_{ik}^-, \tag{11}$$

and diminishing the force arm to zero

$$u_i(\mathbf{x}, t) = \lim_{d\xi_l \to 0}\left(f_k \mathrm{d}\xi_l * \frac{G_{ik}^+ - G_{ik}^-}{\mathrm{d}\xi_l}\right)$$
$$= m_{kl} * \lim_{d\xi_l \to 0}\left(\frac{G_{ik}^+ - G_{ik}^-}{\mathrm{d}\xi_l}\right). \tag{12}$$

Hence,

$$u_i(\mathbf{x}, t) = m_{kl} * G_{ik,l}, \tag{13}$$

where

$$G_{ik,l} = \lim_{d\xi_l \to 0}\left(\frac{G_{ik}^+ - G_{ik}^-}{\mathrm{d}\xi_l}\right) \tag{14}$$

is the spatial derivative of the Green's function with respect to the position vector $\boldsymbol{\xi}$ at the source, and

$$m_{kl} = \lim_{d\xi_l \to 0}\left(f_k \mathrm{d}\xi_l\right) \tag{15}$$

is the moment density of force (or the moment density tensor), and $\mathbf{f} = \mathbf{f}^+ = -\mathbf{f}^-$. If we assume dipole forces distributed along fault Σ, the representation theorem (13) is further expressed as follows:

$$u_i(\mathbf{x}, t) = \int_{-\infty}^{\infty} \mathrm{d}\tau \iint_\Sigma m_{kl}(\boldsymbol{\xi}, \tau)\frac{\partial}{\partial \xi_l}G_{ik}(\mathbf{x}, t; \boldsymbol{\xi}, \tau)\,\mathrm{d}\Sigma(\boldsymbol{\xi}). \tag{16}$$

2.3 Moment tensor

Displacement $\mathbf{u}(\mathbf{x}, t)$ generated by an earthquake source can also be expressed in terms of displacement discontinuity $[\mathbf{u}]$ along fault Σ using the Betti's theorem (Aki & Richards 2002, eq. 3.2):

$$u_i(\mathbf{x}, t) = \int_{-\infty}^{\infty} \mathrm{d}\tau \iint_\Sigma [u_n(\boldsymbol{\xi}, \tau)] c_{njkl}\, n_j \frac{\partial}{\partial \xi_l} G_{ik}(\mathbf{x}, t; \boldsymbol{\xi}, \tau)\, \mathrm{d}\Sigma(\boldsymbol{\xi}), \tag{17}$$

where $\mathbf{n}$ is the normal to the fault Σ. Comparing eqs (16) and (17) we readily obtain for the moment density tensor m_{kl}

$$m_{kl} = c_{ijkl}\, [u_i]\, n_j. \tag{18}$$

For a point source approximation, we define seismic moment tensor $\mathbf{M}$ as

$$M_{kl} = \iint_\Sigma m_{kl}\, d\Sigma = S c_{ijkl}\, [u_i]\, n_j, \tag{19}$$

where S is the fault size, c_{ijkl} are the average elastic parameters along the fault, $[\mathbf{u}]$ is the average displacement discontinuity along the fault, and $\mathbf{n}$ is the normal of the (planar) fault.

3 REPRESENTATION THEOREM FOR SOURCES AT A MATERIAL INTERFACE

3.1 Single body forces

Displacement $\mathbf{u}(\mathbf{x}, t)$ produced by a single force at a contact of two media is obtained in the same way as in the smooth medium

$$u_i = f_n * G_{in}, \tag{20}$$

where the appropriate Green's function must be taken into account. Similarly as in the smoothly inhomogeneous medium, the Green's function for the source at an interface is uniquely defined and is continuous when the source crosses the material interface (Heaton & Heaton 1989a; Jílek & Červený 1996). However, the spatial derivatives of the Green's function are, in general, discontinuous at the interface (Heaton & Heaton 1989a), and consequently, the representation theorem for dipole sources at the interface and the expression for the moment tensor are more involved.

3.2 Dipole body forces

Displacement $\mathbf{u}(\mathbf{x}, t)$ produced by a dipole force at a material interface can be expressed similarly as in smooth media as the sum of displacements $\mathbf{u}^+(\mathbf{x}, t)$ and $\mathbf{u}^-(\mathbf{x}, t)$, produced by individual single forces $\mathbf{f}^+(\boldsymbol{\xi}^+, \tau)$ and $\mathbf{f}^-(\boldsymbol{\xi}^-, \tau)$, forming the dipole force, $\mathbf{f}^+ = -\mathbf{f}^-$ (see Fig. 1b):

$$u_i(\mathbf{x}, t) = u_i^+(\mathbf{x}, t) + u_i^-(\mathbf{x}, t) = f_k^+ * G_{ik}^+ + f_k^- * G_{ik}^-, \tag{21}$$

and diminishing the force arm to zero

$$\begin{aligned} u_i(\mathbf{x}, t) &= \lim_{\mathrm{d}\xi_l \to 0} \left(f_k \mathrm{d}\xi_l * \frac{G_{ik}^+ - G_{ik}^-}{\mathrm{d}\xi_l} \right) \\ &= m_{kl} * \lim_{\mathrm{d}\xi_l \to 0} \left(\frac{G_{ik}^+ - G_{ik}^-}{\mathrm{d}\xi_l} \right), \end{aligned} \tag{22}$$

where $m_{kl} = \lim_{\mathrm{d}\xi_l \to 0} \left(f_k \mathrm{d}\xi_l \right)$ is the moment density tensor, and $\mathbf{f} = \mathbf{f}^+ = -\mathbf{f}^-$. Since some of the spatial derivatives of the Green's function are, in general, discontinuous at interface Σ (Heaton & Heaton 1989a) but continuous outside the interface, we can write

$$\begin{aligned} \lim_{\mathrm{d}\xi_l \to 0} \left(\frac{G_{ik}^+ - G_{ik}^-}{\mathrm{d}\xi_l} \right) &= \lim_{\mathrm{d}\xi_l \to 0} \left(\frac{1}{2} \frac{\left(G_{ik}^+ - G_{ik}^\Sigma\right) + \left(G_{ik}^\Sigma - G_{ik}^-\right)}{\mathrm{d}\xi_l/2} \right) \\ &= \frac{1}{2} \left(G_{ik,l}^+ + G_{ik,l}^- \right), \end{aligned} \tag{23}$$

where G_{ik}^Σ is the Green's function at the interface, and the spatial derivatives of the Green's function $G_{ik,l}^+$ and $G_{ik,l}^-$ are the limits of the derivatives calculated with respect to the position vector $\boldsymbol{\xi}$ at the source from both sides of interface Σ.

Finally, the generalized representation theorem for sources at the material interface reads

$$u_i(\mathbf{x}, t) = m_{kl} * \langle G_{ik,l} \rangle, \tag{24}$$

where

$$\langle G_{ik,l} \rangle = \frac{1}{2} \left(G_{ik,l}^+ + G_{ik,l}^- \right) \tag{25}$$

is the average spatial derivative of the Green's function at the interface. Obviously, eq. (25) holds also for continuous spatial derivatives of the Green's function across the interface (i.e. for the derivatives with respect to coordinates parallel to the material interface).

4 MOMENT TENSOR OF SOURCES NEAR AND AT A MATERIAL INTERFACE

4.1 Discontinuity of the moment tensor across the interface

Let us assume three earthquake sources characterized by the same displacement discontinuity $[\mathbf{u}]$. Two sources will be situated at the first and second half-spaces, respectively, but infinitesimally close to the material interface. The third source will be situated exactly at the interface. Since the sources are characterized by the same displacement discontinuity $[\mathbf{u}]$ and the source positions are infinitesimally close each to the other, all three sources produce the same displacement field $\mathbf{u}(\mathbf{x}, t)$. Taking into account representation theorems (13) and (24), we can write:

$$u_i(\mathbf{x}, t) = m_{kl}^+ * G_{ik,l}^+ = m_{kl}^- G_{ik,l}^- = m_{kl} * \langle G_{ik,l} \rangle, \tag{26}$$

where m_{kl}^+ and m_{kl}^- are the moment density tensors of sources situated at the first and second half-spaces, respectively,

$$m_{kl}^+ = c_{ijkl}^+\, [u_i]\, n_j, \tag{27}$$

$$m_{kl}^- = c_{ijkl}^-\, [u_i]\, n_j, \tag{28}$$

and m_{kl} is the moment density tensor of the source at the interface. Eqs (27) and (28) imply that the moment tensors display jumps when the source crosses the material interface. This observation seems apparently surprising but it reflects a simple fact that the moment tensors represent equivalent body forces but not equivalent tractions at the fault. Although the moment tensors are different for the three considered sources, their tractions are identical. The condition of continuity of traction across the material interface will be used in the next section for deriving the moment density tensor m_{kl} of the source at the interface.

4.2 Moment tensor for a source at the material interface

Adopting a local coordinate system with fault normal $\mathbf{n}$ along the x_3-axis, the Green's functions $\mathbf{G}^+$ and $\mathbf{G}^-$ in eq. (26) must satisfy the following conditions:

$$G^+_{ik,1} = G^-_{ik,1}, \quad G^+_{ik,2} = G^-_{ik,2}, \quad \text{where } i,\ k = 1, 2, 3, \tag{29}$$

and

$$\begin{aligned} c^+_{13kl} G^+_{ik,l} &= c^-_{13kl} G^-_{ik,l}, \quad c^+_{23kl} G^+_{ik,l} = c^-_{23kl} G^-_{ik,l}, \\ c^+_{33kl} G^+_{ik,l} &= c^-_{33kl} G^-_{ik,l}, \qquad \text{where } i = 1, 2, 3. \end{aligned} \tag{30}$$

The conditions express the continuity of the displacement and traction across the material interface. Using these conditions and eq. (26) we can uniquely determine moment tensor m_{kl}. The equations to be solved are analogous to those known in theory of effective media. Applying the averaging theory of Schoenberg & Muir (1989), we obtain for the moment density tensor m_{kl}

$$m_{kl} = \bar{c}_{ijkl} [u_i] n_j, \tag{31}$$

where $\bar{c}_{ijkl}$ are the effective elastic parameters calculated from c^+_{ijkl} and c^-_{ijkl}. If we express elastic parameters c_{ijkl} in the 2-index Voigt notation and combine them into submatrices C_{TT}, C_{NT} and C_{NN} in the following way:

$$C = \begin{bmatrix} C_{TT} & C_{TN} \\ C_{NT} & C_{NN} \end{bmatrix}, \tag{32}$$

where

$$C_{TT} = \begin{bmatrix} c_{11} & c_{12} & c_{16} \\ c_{12} & c_{22} & c_{26} \\ c_{16} & c_{26} & c_{66} \end{bmatrix}, \quad C_{TN} = \begin{bmatrix} c_{13} & c_{14} & c_{15} \\ c_{23} & c_{24} & c_{25} \\ c_{36} & c_{46} & c_{56} \end{bmatrix},$$

$$C_{NN} = \begin{bmatrix} c_{33} & c_{34} & c_{35} \\ c_{34} & c_{44} & c_{45} \\ c_{35} & c_{45} & c_{55} \end{bmatrix}, \quad C_{NT} = C^T_{TN}, \tag{33}$$

the effective elastic parameters finally read (Carcione *et al.* 2012)

$$\begin{aligned} \bar{C}_{NN} &= \left\langle C^{-1}_{NN} \right\rangle^{-1}, \\ \bar{C}_{TN} &= \left\langle C_{TN} C^{-1}_{NN} \right\rangle \bar{C}_{NN}, \\ \bar{C}_{TT} &= \langle C_{TT} \rangle - \left\langle C_{TN} C^{-1}_{NN} C_{NT} \right\rangle + \bar{C}_{TN} \left\langle C^{-1}_{NN} C_{NT} \right\rangle. \end{aligned} \tag{34}$$

Brackets $\langle\cdot\rangle$ in (34) mean averaging over the first and second half-space, for example,

$$\begin{aligned} \langle C_{TT} \rangle &= \frac{1}{2} \left(C^+_{TT} + C^-_{TT} \right), \\ \left\langle C^{-1}_{NN} \right\rangle &= \frac{1}{2} \left[\left(C^+_{NN} \right)^{-1} + \left(C^-_{NN} \right)^{-1} \right]. \end{aligned} \tag{35}$$

5 SHEAR AND TENSILE FAULTING AT A CONTACT OF TWO TRANSVERSELY ISOTROPIC MEDIA AND TWO ISOTROPIC MEDIA

5.1 Transversely isotropic media

The averaging formulae (34) can be specified for faulting in transversely isotropic media. Adopting the local coordinate system in which the fault normal is along the x_3-axis, $\mathbf{n} = (0, 0, 1)^T$, we obtain (Backus 1962):

$$\begin{aligned} \bar{c}_{11} &= \bar{c}_{22} = \langle c_{11} \rangle - \left\langle c^2_{13} c^{-1}_{33} \right\rangle + \left\langle c_{13} c^{-1}_{33} \right\rangle^2 \left\langle c^{-1}_{33} \right\rangle^{-1}, \\ \bar{c}_{12} &= \bar{c}_{11} - 2\bar{c}_{66},\ \bar{c}_{13} = \bar{c}_{23} = \left\langle c_{13} c^{-1}_{33} \right\rangle \left\langle c^{-1}_{33} \right\rangle^{-1}, \\ \bar{c}_{33} &= \left\langle c^{-1}_{33} \right\rangle^{-1},\ \bar{c}_{44} = \bar{c}_{55} = \left\langle c^{-1}_{44} \right\rangle^{-1},\ \bar{c}_{66} = \langle c_{66} \rangle, \end{aligned} \tag{36}$$

the other elastic parameters being zero.

Subsequently, the moment tensor of a point pure tensile source with displacement discontinuity $[\mathbf{u}] = (0, 0, u)^T$ reads

$$\mathbf{M}^{\text{tensile}} = uS \begin{bmatrix} \left\langle c_{13} c^{-1}_{33} \right\rangle \left\langle c^{-1}_{33} \right\rangle^{-1} & 0 & 0 \\ 0 & \left\langle c_{13} c^{-1}_{33} \right\rangle \left\langle c^{-1}_{33} \right\rangle^{-1} & 0 \\ 0 & 0 & \left\langle c^{-1}_{33} \right\rangle^{-1} \end{bmatrix}, \tag{37}$$

and the moment tensor of a point shear source with displacement discontinuity $[\mathbf{u}] = (u, 0, 0)^T$ reads

$$\mathbf{M}^{\text{shear}} = uS \begin{bmatrix} 0 & 0 & \left\langle c^{-1}_{44} \right\rangle^{-1} \\ 0 & 0 & 0 \\ \left\langle c^{-1}_{44} \right\rangle^{-1} & 0 & 0 \end{bmatrix}. \tag{38}$$

5.2 Isotropic media

In isotropic media, defined by the Lamé coefficient λ and μ

$$c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right), \tag{39}$$

where δ_{ij} is the Kronecker delta, the effective elastic parameters read

$$\begin{aligned} \bar{c}_{11} &= \bar{c}_{22} = \langle \lambda + 2\mu \rangle - \left\langle \lambda^2 (\lambda + 2\mu)^{-1} \right\rangle \\ &\quad + \left\langle \lambda (\lambda + 2\mu)^{-1} \right\rangle^2 \left\langle (\lambda + 2\mu)^{-1} \right\rangle^{-1},\ \bar{c}_{12} = \bar{c}_{11} - 2\bar{c}_{66}, \\ \bar{c}_{13} &= \bar{c}_{23} = \left\langle \lambda (\lambda + 2\mu)^{-1} \right\rangle \left\langle (\lambda + 2\mu)^{-1} \right\rangle^{-1},\ \bar{c}_{33} = \left\langle (\lambda + 2\mu)^{-1} \right\rangle^{-1}, \\ \bar{c}_{44} &= \bar{c}_{55} = \left\langle \mu^{-1} \right\rangle^{-1},\ \bar{c}_{66} = \langle \mu \rangle. \end{aligned} \tag{40}$$

Note that the effective medium is not isotropic but transversely isotropic.

The moment tensors of point tensile and shear sources with displacement discontinuity $[\mathbf{u}] = (0, 0, u)^T$ and $[\mathbf{u}] = (u, 0, 0)^T$, respectively, are expressed as

$$\mathbf{M}^{\text{tensile}} = uS \begin{bmatrix} \dfrac{\left\langle \lambda (\lambda + 2\mu)^{-1} \right\rangle}{\left\langle (\lambda + 2\mu)^{-1} \right\rangle} & 0 & 0 \\ 0 & \dfrac{\left\langle \lambda (\lambda + 2\mu)^{-1} \right\rangle}{\left\langle (\lambda + 2\mu)^{-1} \right\rangle} & 0 \\ 0 & 0 & \dfrac{1}{\left\langle (\lambda + 2\mu)^{-1} \right\rangle} \end{bmatrix}, \tag{41}$$

$$\mathbf{M}^{\text{shear}} = uS \begin{bmatrix} 0 & 0 & \left\langle \mu^{-1} \right\rangle^{-1} \\ 0 & 0 & 0 \\ \left\langle \mu^{-1} \right\rangle^{-1} & 0 & 0 \end{bmatrix}. \tag{42}$$

Consequently, the scalar seismic moment M_0 for shear sources reads

$$M_0 = 2\frac{\mu^+\mu^-}{\mu^+ + \mu^-}\,[u]\,S. \tag{43}$$

This formula has been proposed by Wu & Chen (2003) using a simplified derivation based on physical understanding of the problem and on a modified definition of the moment tensor. In contrast, our approach is mathematical, eq. (43) is derived using the standard definition of the moment tensor, and it is shown that this equation is unique without any ambiguity.

6 DISCUSSION

Let us shortly discuss why Ampuero & Dahlen (2005) incorrectly stated that the moment tensors of sources at a material interface are ambiguous and concluded that the description using source (potency) tensors must be used. I will focus on this paper because this paper is most comprehensive and the arguments in the other papers are similar.

The authors assert (Ampuero & Dahlen 2005, p. 393) that the 'prescribed slip Δu_k is equivalent either to a superposition of double couples situated on the front side of the fault. . . ' (in our notation, on Σ^+) or 'to a superposition of double couples situated on the back side of the fault. . . ' (in our notation, on Σ^-). This assumption is physically as well as mathematically incorrect and arises from misunderstanding of meaning of the moment tensors. The moment tensors always describe force couples acting on two respective sides of a fault: force $\mathbf{f}^+$acting on Σ^+ (and producing displacement $\mathbf{u}^+$) and force $\mathbf{f}^-$ acting on Σ^- (and producing displacement $\mathbf{u}^-$), and the fault can either lie in the first or second half-spaces or at their contact Σ. So we cannot mix all three cases.

The statement about the ambiguity of the moment tensor at the material interface is unacceptable also for the following reason. A problem of generating displacement field $\mathbf{u}(\mathbf{x}, t)$ by dipole body forces acting at the material interface is well defined from mathematical as well as physical points of view and it must have a unique solution. This excludes any ambiguity.

Let us consider a source characterized by the same displacement discontinuity, which moves across a material interface. As mentioned in Section 4, the moment tensor displays jumps. First, when the source moves from one half-space to the interface, and second, when the source moves from the interface to the other half-space. These jumps are somewhat surprising and undesirable but they reflect the fact that the moment tensors represent equivalent body forces but not equivalent tractions at the fault. The jumps of body forces are physical and cannot be interpreted as an ambiguity of the moment tensor. We cannot deduce that the moment tensor is an ambiguous quantity just because the moment tensor inversion is sensitive to the source position or to the elastic parameters at the source area. Consequently, we conclude that the moment tensors of sources near or at the material interface are unique and can be determined provided we know the source positions and elastic properties of the medium.

However, Ampuero & Dahlen (2005) are correct when claiming that the behaviour of source tensors is essentially different from that of moment tensors. While the moment tensors change discontinuously across the material interface for the source characterized by the same displacement discontinuity, the source tensors change smoothly with no jumps at the interface. Thus a direct inversion for the source tensors might be more stable and of higher accuracy than that for the moment tensors if the fault is close or at the material interface. Obviously, the actual advantages of the source tensor inversion should be proved in future numerical studies.

7 CONCLUSION

The moment tensor is a unique quantity describing the rupture process in any medium including faulting at a material interface. The moment tensor cannot be viewed as an abundant or ambiguous description, which can be replaced by the source (potency) tensor. The moment tensor describes equivalent body forces at the source, which are responsible for the radiation of waves and for static deformations. The source tensor describes just the deformation at the source, that is, the product of acting forces. Although the moment tensor is far from describing actual forces at an earthquake source it is still an important quantity broadly used until a more appropriate force description of the earthquake source will be found.

If the fault lies at a material interface, some of the spatial derivatives of the Green's function needed in the representation theorem are discontinuous and the representation theorem must be modified. If the moment tensor is inverted using the generalized representation theorem (24) with appropriate Green's functions for a source situated at the material interface (Ben-Zion 1990; Jílek & Červený 1996) and using high-quality data, the inversion must yield a unique moment tensor. Ambiguities in moment tensors arise if the velocity model is not well known, the sources are mislocated, Green's functions are inappropriate, or observations are limited or restricted to a narrow frequency band. These deficiencies project into the inaccuracy of moment tensors. The inaccuracy can be particularly significant if the source area is strongly heterogeneous, because the moment tensors are sensitive to material properties at the source and a small location error can produce significantly different moment tensors.

Shear faulting at a planar material interface in isotropic media is represented by the standard double-couple moment tensor. The scalar seismic moment is calculated as a product of the displacement discontinuity across the fault, the fault size and the effective rigidity at the fault. The effective rigidity is calculated as the harmonic mean of rigidities at the individual sides of the fault.

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