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Energy Balance of Simple Elastodynamic Sources

VÁCLAV VAVRYČUK[1,2]

Abstract—Complete relations are derived for energy and energy flux of elastic waves generated by an isotropic and double-couple source in a perfectly elastic, homogeneous, isotropic, and unbounded medium. In the energy balance of elastodynamic sources near-field waves play an essential role, transforming static energy into wave energy, and *vice versa.* For explosive and dislocation sources, the source surface radiates a positive wave energy that is partially distributed to the medium transforming into static energy. For implosive and antidislocation sources, the source surface generates elastic waves, but it does not necessarily imply that it also radiates a positive wave energy. The energy transported by waves can originate in gradual transformation of the static-to-wave energy during propagation of waves through a stressed medium.

Key words: Double-couple, earthquake, elastodynamics, energy balance, isotropic source, near-field waves, wave energy.

1. Introduction

REID (1910) mentioned, for the first time, the idea of an earthquake as a process of sudden release of static strain energy, accumulated extendedly in the medium surrounding a fault. According to this concept, the released energy is partially expended in breaking up material and crack growth on the one hand, and radiated away into a distant medium on the other. Each part of energy is transported by seismic waves: 1) by near-field waves from a surrounding medium to the fault; and 2) by far-field waves from the surrounding medium into a distant medium. In deliberations of *energy balance of earthquake sources*, the energy transported by both wave types (i.e., the complete elastodynamic solution) should be taken into account (KOSTROV, 1974; HUSSEINI *et al.*, 1975; KOSTROV and DAS, 1989; FREUND, 1990). However, the energy balance of earthquake sources is quite complex, preventing the study of near-field and far-field wave energy separately and providing a clear physical interpretation of individual terms. To simplify the problem, the

[1] Present address: Department of Earth and Planetary Systems Science, Hiroshima University, 1-3-1 Kagami-yama, Higashi-Hiroshima 724, Japan.

[2] On leave from: Geophysical Institute, Czech Academy of Sciences, Boční II/1401, 141 31 Praha 4, Czech Republic.

near-field wave energy can be neglected as is widely done in works on *radiated energy* (RANDALL, 1973; BOATWRIGHT, 1980; RUDNICKI and FREUND, 1981; VASSILIOU and KANAMORI, 1982; BOATWRIGHT and FLETCHER, 1984; BOATWRIGHT and CHOY, 1986; MADARIAGA, 1986; KIKUCHI and FUKAO, 1988). This approach can produce reasonable estimates of the energy transported by a wavefield for a wide range of distances from a source. In this paper we will follow another approach to the energy balance of wavefields. It applies to all distances, enabling us to study the energy of far-field waves as well as near-field waves and to thoroughly discuss their physical properties and mutual relations. We will simplify the problem, adopting basic sources in a plain medium: *isotropic* or *double-couple sources* situated in a *perfectly elastic, homogeneous, isotropic, unbounded* and *prestress free medium*. The first source is the simplest in elastodynamics and also was used by many other authors (ESHELBY, 1957; YOSHIYAMA, 1963; RANDALL, 1964; RUDNICKI, 1983); the second source is the most frequently applied in seismology. Since an exact elastodynamic solution exists for each source (ACHENBACH, 1975; ERINGEN and SUHUBI, 1975; AKI and RICHARDS, 1980), the problem of energy flow can be solved analytically. Physical interpretation of the energy of near-field and far-field waves is therefore straightforward and well-comprehensible and can also offer an insight into the energy balance of wavefields generated by more complex sources such as a real earthquake source.

2. *Notation*

The following list presents the notation used in this paper. Physical units for selected quantities are given in brackets.

$h(\vec{x}, t)$	elastic energy density [J/m^3]
$h_K(\vec{x}, t)$, $h_U(\vec{x}, t)$	kinetic and potential elastic energy densities
$\vec{p}(\vec{x}, t)$	energy flux density [W/m^2]
$H(S, t)$	surface elastic energy distributed over the surface S [J/m]
$H_K(S, t)$, $H_U(S, t)$	kinetic and potential surface elastic energies
H^S, H^D, H^T, H^R	static, dynamic, transient and residual parts of the surface energy $H(S, t)$
$P(S, t)$	energy flux over the surface S [W]
P^D, P^T, P^R	dynamic, transient and residual parts of the energy flux $P(S, t)$
$E(V, t)$	elastic energy contained in the volume V [J]
$E_\varepsilon(V, t)$	elastic energy contained in the volume V outside of a small spherical cavity at the origin of coordinates with radius ε
$T(t)$	total elastic energy contained in the medium [J]
$T_\varepsilon(t)$	total elastic energy in the medium outside of a small spherical cavity at the origin of coordinates
$T_{\varepsilon 0}$, $T_{\varepsilon 1}$	initial and final values of the total elastic energy $T_\varepsilon(t)$

T_ε^S, T_ε^D, T_ε^T, T_ε^R	static, dynamic, transient and residual parts of $T_\varepsilon(t)$
$W(V)$	wave elastic energy flowing out of the volume V [J]
W^D, W^T, W^R	dynamic, transient and residual parts of the wave energy $W(V)$
$M_{ij}(t)$	moment tensor
$M(t)$	source-time function
M_0, M_1	initial and final values of $M(t)$
$M_\alpha = M(t - r/\alpha)$,	
$M_\beta = M(t - r/\beta)$	retarded source-time functions
T	source process duration
α, β	P- and S-wave velocities
$\lambda^P = \alpha T$	wavelength of the P wave
c_{ijkl}, λ, μ, ρ	elasticity tensor, Lamé's constants, density of the medium
$\vec{u}(\vec{x}, t)$	displacement vector
$\tau_{ij}(\vec{x}, t)$	stress tensor
$e_{ij}(\vec{x}, t)$	(infinitesimal) strain tensor
δ_{ij}	Kronecker delta
$\delta(t)$	Dirac delta function
ε	radius of the small cavity radiating elastic waves
$\vec{\gamma}$	direction vector of an observer
r	distance of an observer from the source

If the volume is a sphere with radius r, the variable r is used in place of variables V or S.

3. *Definitions and Basic Formulae*

Let us assume a linearly elastic medium. The *elastic energy density* $h(\vec{x}, t)$ and the *energy flux density* $\vec{p}(\vec{x}, t)$ can be expressed as follows (BEN-MENAHEM and SINGH, 1981)

$$h(\vec{x}, t) = h_K + h_U = \tfrac{1}{2}\rho\dot{u}_i\dot{u}_i + \tfrac{1}{2}\tau_{ij}e_{ij} = \tfrac{1}{2}\rho\dot{u}_i\dot{u}_i + \tfrac{1}{2}c_{ijkl}u_{i,j}u_{k,l}, \tag{1}$$

$$p_i(\vec{x}, t) = -\tau_{ij}\dot{u}_j = -c_{ijkl}u_{k,l}\dot{u}_j, \tag{2}$$

where $h_K(\vec{x}, t)$ and $h_U(\vec{x}, t)$ are the kinetic and potential (strain) energy densities, $\tau_{ij}(\vec{x}, t)$ denotes the stress tensor, c_{ijkl} is the elasticity tensor, $e_{ij}(\vec{x}, t)$ is the strain tensor, $\vec{u}(\vec{x}, t)$ is the displacement vector, and ρ is the density of the medium. Dots over quantities mean time derivatives, indices after the comma denote spatial derivatives. The Einstein summation convention is applied. If we assume the absence of any energy sources in the medium, the energy flux density is related to the elastic energy density as

$$p_{i,i} + \frac{\partial h}{\partial t} = 0, \tag{3}$$

which expresses the *local conservation of energy*. Assuming a volume V is bounded by a surface S and integrating formulae (1) and (2) over S, we will introduce *surface elastic energy* $H(S, t)$ and *energy flux* $P(S, t)$

$$H(S, t) = \iint_S h(\vec{x}, t)\, dS, \tag{4a}$$

$$P(S, t) = \iint_S p_i(\vec{x}, t) n_i\, dS, \tag{4b}$$

where $\vec{n}$ is the outer normal of the surface S. Consequently, *elastic energy* $E(V, t)$ contained in the volume V, *total elastic energy* $T(t)$ of the whole elastic solid, and *wave energy* $W(V)$ flowing out of the volume V over the whole time history, are defined

$$E(V, t) = \iiint_V h(\vec{x}, t)\, dV, \tag{5a}$$

$$T(t) = E(V_\infty, t) = \iiint_{V_\infty} h(\vec{x}, t)\, dV, \tag{5b}$$

$$W(V) = \int_0^\infty P(S, t)\, dt, \tag{5c}$$

where V_∞ is the volume of a whole elastic solid.

4. *Isotropic Source*

4.1. Displacement Field

Let us assume an isotropic point source at the origin of coordinates, so that the moment tensor has the form

$$M_{ij}(t) = \delta_{ij} M(t),$$

where $M(t)$ is the source-time function. Let the source be situated in an isolated, perfectly elastic, homogeneous, isotropic, and unbounded medium. The exact solution of the elastodynamic equation reads (ACHENBACH, 1975, p. 102)

$$u_i(\vec{x}, t) = \frac{\gamma_i}{4\pi\rho\alpha^2} \frac{M(t - r/\alpha)}{r^2} + \frac{\gamma_i}{4\pi\rho\alpha^3} \frac{\dot{M}(t - r/\alpha)}{r}, \tag{6}$$

where $\vec{\gamma}$ denotes the direction vector of an observer, r the distance of an observer from the source, and α is the P-wave velocity.

The first and second terms in (6) are called *near-field P waves* (P^N) and *far-field P waves* (P^F), respectively. No S waves are generated by this source. If $M(t)$ is time-independent, the elastodynamic equation (equation of motion) reduces to an elastostatic equation (equation of equilibrium), and the solution consists only of the first term in (6). Accordingly, the near-field waves are related to the static field or more exactly, the near-field waves are responsible for any change of the static field. The amplitude of the P^N waves decreases as $1/r^2$, while that of the P^F waves as $1/r$. The P^N and P^F waves are polarized linearly in the ray direction and have spherically symmetric radiation patterns.

To avoid a singularity of physical quantities at the point source, we will consider a source of finite size: a small spherical cavity with radius ε, its center being at the origin of coordinates. If formula (6) is to describe the displacement field for the cavity as well, the displacement generated by the point source at distance, $r = \varepsilon$ must coincide with the displacement at the cavity surface. As a consequence, the source process on the cavity surface is retarded by time ε/α, and the time dependence of the cavity surface displacement is not identical with the source-time function $M(t)$ (see Fig. 2), although it obviously converges to $M(t)$ as $\varepsilon \to 0$.

We will consider two forms of source-time function $M(t)$ (Fig. 1) being referred to as the explosive and implosive sources. For the *explosive source*, $M(t)$ is increasing from $M_0 = M(t \leq 0) = 0$ to $M_1 = M(t \geq T) > 0$, where T is the source process duration. That means that the medium is initially free of stress and deformations. During the source process, the source expands in response to external forces applied to its surface. After the source process terminates the external forces hold the source in an expanded state. The source expansion generates elastic waves and effects a transition from an unstressed to a stressed state of a medium. For the *implosive source*, $M(t)$ decreases from $M_0 > 0$ to $M_1 = 0$, implying that the source is

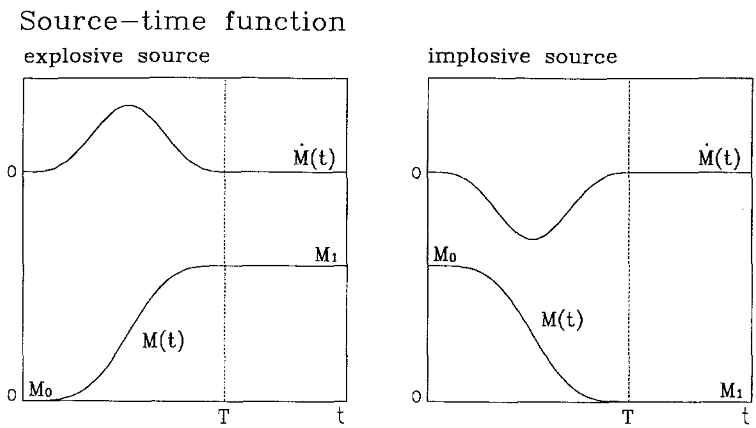

Figure 1
Source-time function $M(t)$ and its time derivative for explosive and implosive sources. The vertical dotted line denotes the end of the source process.

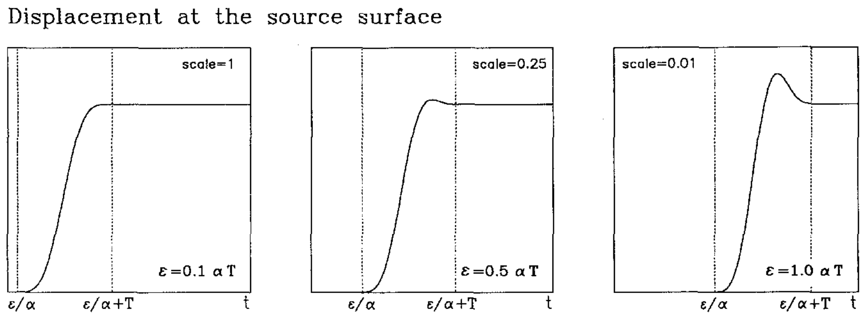

Figure 2

Radial displacement at the source surface for the spherical expanding cavity as a function of time and cavity radius ε, using formula (6). The vertical dotted lines denote the beginning and the end of the source process.

in an expanded state initially, the medium being stressed around the source. No prestress or other stress sources are considered to exist in a medium. During the source process, the source contracts in response to the weakening or ceasing of external forces at its surface. The implosion generates elastic waves and totally relaxes the stress in the medium. If the forces are suddenly interrupted, then no external forces act during implosion, thus the process is spontaneous. We call this source an *inner* source, because during spontaneous implosion no external power is supplied into the medium, thus the total energy in the medium conserves.

4.2. Energy and Energy Flux

Formula (6) for the elastodynamic field generated by a spherical cavity gives a possibility to express all energy quantities in an exact analytical form, and consequently to solve exactly the energy balance of the complete wavefield. A derivation of $h(\vec{x}, t)$ and $\vec{p}(\vec{x}, t)$ is tedious, but elementary. Also it can be verified that the local energy conservation law (3) holds for solution (6). Next, only formulae for surface energy $H(r, t)$ on the sphere S with radius r, its center being at the origin of coordinates, and energy flux $P(r, t)$ over S will be presented

$$H(r, t) = \frac{\mu}{4\pi\rho^2}\left\{\frac{6}{\alpha^4}\frac{M_\alpha^2}{r^4} + \frac{12}{\alpha^5}\frac{M_\alpha \dot{M}_\alpha}{r^3} + \frac{4}{\alpha^6}\frac{M_\alpha \ddot{M}_\alpha}{r^2} + \frac{6}{\alpha^6}\frac{\dot{M}_\alpha^2}{r^2} + \frac{4}{\alpha^7}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r}\right\}$$

$$+ \frac{\lambda + 2\mu}{4\pi\rho^2}\left\{\frac{1}{2\alpha^6}\frac{\dot{M}_\alpha^2}{r^2} + \frac{1}{\alpha^7}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r} + \frac{1}{\alpha^8}\ddot{M}_\alpha^2\right\}, \tag{7}$$

$$P(r, t) = \frac{\mu}{4\pi\rho^2}\left\{\frac{4}{\alpha^4}\frac{M_\alpha \dot{M}_\alpha}{r^3} + \frac{4}{\alpha^5}\frac{M_\alpha \ddot{M}_\alpha}{r^2} + \frac{4}{\alpha^5}\frac{\dot{M}_\alpha^2}{r^2} + \frac{4}{\alpha^6}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r}\right\}$$

$$+ \frac{\lambda + 2\mu}{4\pi\rho^2}\left\{\frac{1}{\alpha^6}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r} + \frac{1}{\alpha^7}\ddot{M}_\alpha^2\right\}, \tag{8}$$

where $M_\alpha = M(t - r/\alpha)$. Formulae (7) and (8) are complex, because they consist not only of far-field and near-field terms, but also of interaction terms between the far-field and near-field waves. Because of complexity of energy balance described by (7) and (8), we will study integral energy quantities at first (Sec. 4.3.), and then the individual terms of the energy and energy flux will be discussed in detail (Sec. 4.4.).

4.3. Total and Wave Energy

Total energy can be expressed by integrating (7) over r (from ε to ∞). Before the source process begins, $t \leq \varepsilon/\alpha$, we have

$$T_{\varepsilon 0} = T_\varepsilon(t \leq \varepsilon/\alpha) = \frac{\mu}{4\pi\rho^2\alpha^4}\frac{2M_0^2}{\varepsilon^3}, \tag{9}$$

and after the source process terminates, $t \geq \varepsilon/\alpha + T$,

$$T_{\varepsilon 1} = T_\varepsilon(t \geq \varepsilon/\alpha + T) = \frac{1}{4\pi\rho^2\alpha^4}\left\{\mu\frac{2M_1^2}{\varepsilon^3} + \frac{\lambda + 2\mu}{\alpha^3}\int_0^T \ddot{M}^2(t)\,dt\right\}, \tag{10}$$

where T_ε denotes the total energy in the medium beyond the cavity. For the *explosive source*, initial total elastic energy $T_{\varepsilon 0}$ equals zero (Fig. 3); $T_\varepsilon(t)$ increases with time, during which the external forces at the source surface supply energy into the medium. When the source process terminates ($t = \varepsilon/\alpha + T$), $T_\varepsilon(t)$ reaches final value $T_{\varepsilon 1}$, being constant thereafter. The final value is the sum of the static energy (first term in (10)) and of the energy of the far-field waves (second term in (10)) radiated by the source. For the *implosive source*, $T_{\varepsilon 0}$ equals the static energy previously accumulated in the medium. In the final state, $T_\varepsilon(t)$ equals only the

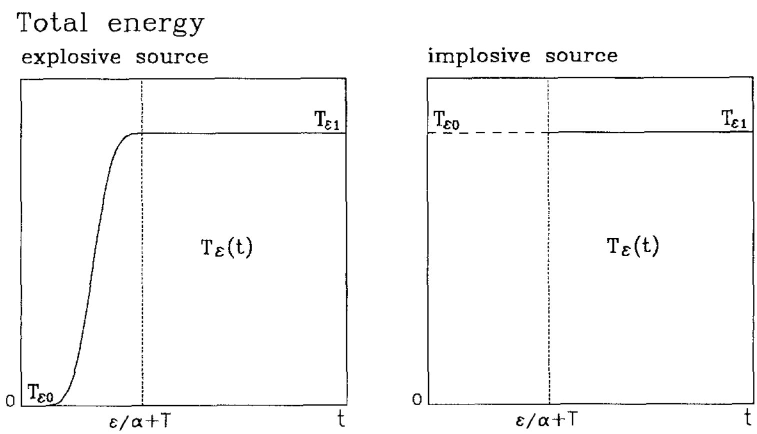

Figure 3
Total energy T_ε as a function of time. The dashed line for times $t \leq \varepsilon/\alpha + T$ (right-hand plot) displays $T_\varepsilon(t)$ during the source process for the inner source. The vertical dotted line denotes the end of the source process.

elastic energy of the far-field waves, because the medium has become unstrained. The $T_{\varepsilon 0}/T_{\varepsilon 1}$ ratio depends on the form of source-time function $M(t)$. For a high-frequency range, $T_{\varepsilon 1}$ is greater than $T_{\varepsilon 0}$, because of dominance of the $\int \ddot{M}^2(t)\,dt$ term, and *vice versa* for a low-frequency range. For an *inner implosive source*, the total energy of a medium conserves, so that $T_{\varepsilon 0} = T_{\varepsilon 1}$ (Fig. 3).

As regards *wave energy* $W(r)$, formula (5c) and (8) yield

$$W(r) = \frac{1}{4\pi\rho^2\alpha^4}\left\{2\mu\frac{M_1^2 - M_0^2}{r^3} + \frac{\lambda + 2\mu}{\alpha^3}\int_0^T \ddot{M}^2(t)\,dt\right\}. \tag{11}$$

The first and second terms in (11) correspond to the wave energies of the near-field and far-field waves, respectively. The near-field wave energy can be either positive or negative, and depends on the distance, vanishing at long distances. At the cavity surface it has exactly the same value as the static energy (formulae (9) and (10)), which is loaded into the medium (explosive source) or released (implosive source) during the wave propagation through the medium. In contrast, the far-field wave energy is always positive and does not depend on the distance. For the *explosive source*, the wave energy $W(r)$ at $r = \varepsilon$ equals the work spent by force at the source surface. $W(r)$ decreases with increasing distance, because the near-field wave energy transforms into static energy. At longest distances, $W(r)$ converges to the wave energy of the far-field waves. For the *implosive source*, $W(r)$ is an increasing function of distance, also converging to the far-field wave energy. The increase of $W(r)$ results from the static-to-wave energy transformation. The value of the wave energy radiated by the source surface depends on the action of external forces upon it. If positive energy is produced at the source surface, then $W(r = \varepsilon)$ is positive, and if the energy is consumed, then wave energy $W(r = \varepsilon)$ is negative. For the *inner implosive source*, no external forces act during the source process, thus, the wave energy $W(r = \varepsilon)$ radiated by the surface equals zero. The zero value of $W(r = \varepsilon)$ can also be obtained directly from (2), if we consider that the surface of the spontaneously implosive source is by definition free of traction.

4.4. *Types of Energy and Energy Flux: Formulae*

In order to understand the time-space evolution of energy, we will split relation (7) for surface energy H into several terms which will represent physically different energy forms:

$$H = H^S + H^D + H^T + H^R, \tag{12a}$$

where H^S, H^D, H^T and H^R will be called the static, dynamic, transient and residual parts of H, respectively,

$$H^S = \frac{6\mu}{4\pi\rho^2\alpha^4}\frac{M_\alpha^2}{r^4}, \tag{12b}$$

$$H^D = \frac{\lambda + 2\mu}{4\pi\rho^2\alpha^8}\ddot{M}_\alpha^2, \tag{12c}$$

$$H^T = \frac{4\mu}{4\pi\rho^2\alpha^5}\frac{M_\alpha \dot{M}_\alpha}{r^3}, \tag{12d}$$

$$H^R = \frac{\mu}{4\pi\rho^2}\left\{\frac{8}{\alpha^5}\frac{M_\alpha \dot{M}_\alpha}{r^3} + \frac{4}{\alpha^6}\frac{M_\alpha \ddot{M}_\alpha}{r^2} + \frac{6}{\alpha^6}\frac{\dot{M}_\alpha^2}{r^2} + \frac{4}{\alpha^7}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r}\right\}$$
$$+ \frac{\lambda + 2\mu}{4\pi\rho^2}\left\{\frac{1}{2\alpha^6}\frac{\dot{M}_\alpha^2}{r^2} + \frac{1}{\alpha^7}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r}\right\}. \tag{12e}$$

Integrating formulae (12) over r we obtain

$$T^S_{\varepsilon 0} = \frac{2\mu}{4\pi\rho^2\alpha^4}\frac{M_0^2}{\varepsilon^3}, \quad T^S_{\varepsilon 1} = \frac{2\mu}{4\pi\rho^2\alpha^4}\frac{M_1^2}{\varepsilon^3}, \tag{13a}$$

$$T^D_\varepsilon(t \geq \varepsilon/\alpha + T) = \frac{\lambda + 2\mu}{4\pi\rho^2\alpha^8}\int_0^T \ddot{M}^2(t)\, dt, \tag{13b}$$

$$T^R_\varepsilon(t \geq \varepsilon/\alpha + T) = 0, \tag{13c}$$

where $T^S_{\varepsilon 0} = T^S_\varepsilon(t = 0)$ and $T^S_{\varepsilon 1} = T^S_\varepsilon(t \to \infty)$ denote the initial and final total static energies; T^D_ε and T^R_ε the total dynamic and residual energies.

Integrating (12b,d) over r for $t \geq \varepsilon/\alpha + T$, we get

$$T^T_\varepsilon(t) + T^S_\varepsilon(t) = \frac{2\mu}{4\pi\rho^2\alpha^4}\frac{M_1^2}{\varepsilon^3} = T^S_{\varepsilon 1}, \tag{14}$$

where T^T_ε and T^S_ε are the total transient and static energies.

Analogously to (12), we will define the dynamic, transient and residual parts of the energy flux P expressed by (8):

$$P = P^D + P^T + P^R, \tag{15a}$$

$$P^D = \frac{\lambda + 2\mu}{4\pi\rho^2\alpha^7}\ddot{M}_\alpha^2, \tag{15b}$$

$$P^T = \frac{4\mu}{4\pi\rho^2\alpha^4}\frac{M_\alpha \dot{M}_\alpha}{r^3}, \tag{15c}$$

$$P^R = \frac{1}{4\pi\rho^2}\left\{\frac{4\mu}{\alpha^5}\frac{M_\alpha \ddot{M}_\alpha}{r^2} + \frac{4\mu}{\alpha^5}\frac{\dot{M}_\alpha^2}{r^2} + \frac{4\mu}{\alpha^6}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r} + \frac{\lambda + 2\mu}{\alpha^6}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r}\right\}. \tag{15d}$$

Integrating (15) over t, we get

$$W^D = \frac{\lambda + 2\mu}{4\pi\rho^2\alpha^7}\int_0^T \ddot{M}^2(t)\, dt, \tag{16a}$$

$$W^T = \frac{2\mu}{4\pi\rho^2\alpha^4}\frac{M_1^2 - M_0^2}{r^3}, \tag{16b}$$

$$W^R = 0, \tag{16c}$$

where W^D, W^T, and W^R are the dynamic, transient and residual wave energies, respectively.

4.5. Types of Energy and Energy Flux: Discussion

Formula (12b) indicates that the *static energy* H^S is a nonlocal, potential energy, that can be distributed through the whole medium, but mostly concentrated near the source, decreasing quickly with the distance (see Fig. 4a). Every loading or release of the static energy is performed by the near-field waves, implying that any

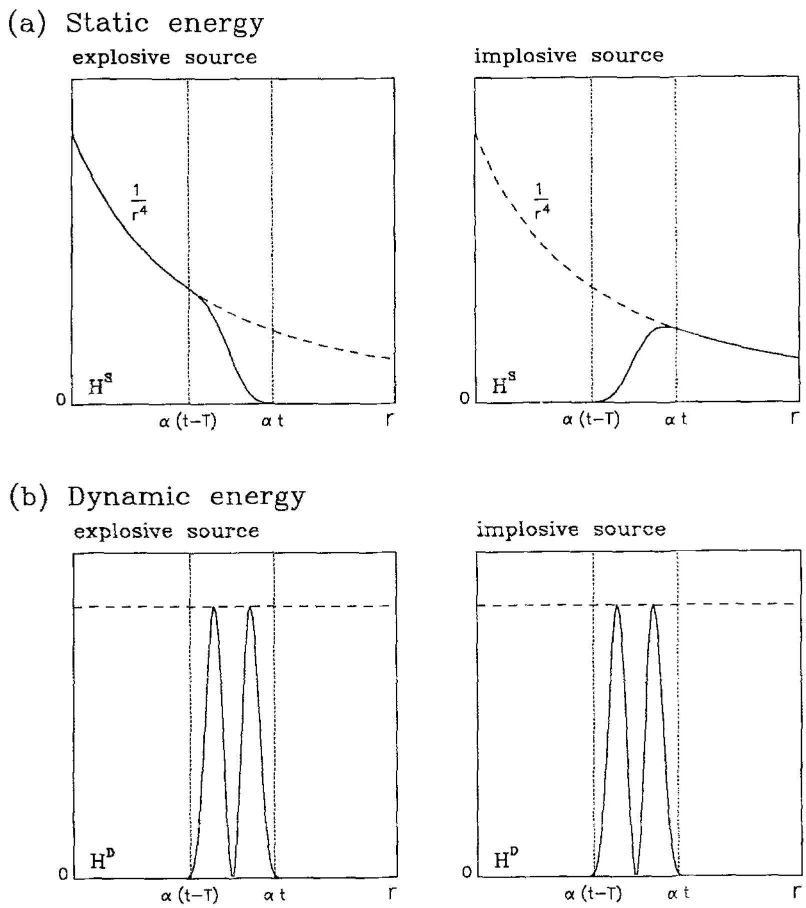

Figure 4
(a) Schematic plot of static surface energy H^S for the isotropic source as a function of distance at a fixed instant t. The vertical dotted lines delineate the space interval of the P wave. The dashed line shows the envelope of H^S for different times. (b) Dynamic surface energy H^D as a function of distance at a fixed instant t. The dashed line shows the envelope of maximum amplitudes of H^D for different times.

change of the static energy is gradual in accordance with the causality principle. No flux and no wave energy terms correspond to the static energy.

From (12c) the *dynamic energy* H^D is local, appearing only in the time-space interval of waves (Fig. 4b). It can be shown that H^D consists equally of potential and kinetic energies. The initial value of total dynamic energy T_ε^D is zero. It increases during the source process and becomes constant at all times after it has terminated (13b). Dynamic wave energy W^D is also constant and does not depend on the distance from the source (16a). This energy corresponds to the usually calculated *radiated (seismic) energy*, to which the far-field approximation or the zero-order approximation of the ray theory is applied. The common use of W^D, instead of $W(r)$, is justified in practical applications because of the fast convergence of W to W^D with distance.

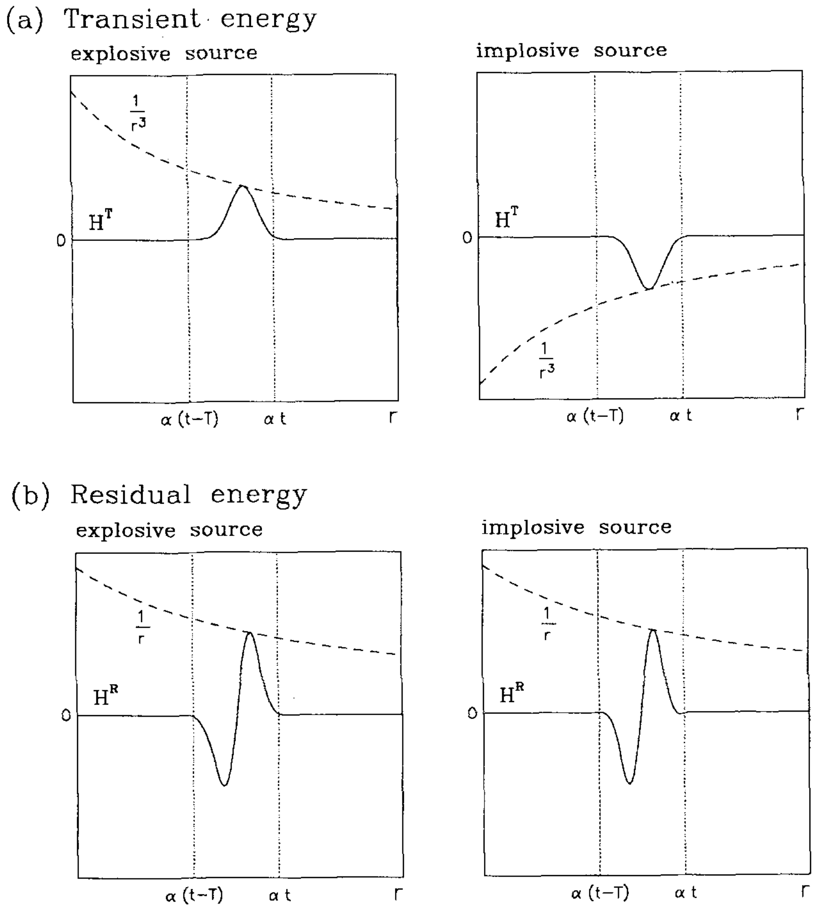

Figure 5
Schematic plot of transient surface energy H^T (a) and residual surface energy H^R (b) for the isotropic source as a function of distance at a fixed instant t.

Formulae (12d) and (14) indicate that the *transient energy* H^T is local (see Fig. 5a), closely related to the static energy. H^T is a potential energy, forming a closed energy system with the static energy, in which both types of energy can be transformed to each other in the course of time. For the *explosive source*, $T_\varepsilon^T(t)$ is produced by external forces during the source process. Once the source process has terminated, no external power is supplied into a medium, and $T_\varepsilon^T(t)$ decreases, transforming into static energy until it vanishes (Fig. 6a). Likewise, $W^T(r)$ is a decreasing function converging to zero as $r \to \infty$ (Fig. 6b). The decrease of $W^T(r)$ is the result of wave-to-static energy transformation related to the loading of static energy in the medium. As regards the *implosive source*, $T_\varepsilon^T(t)$ is negative immediately after the

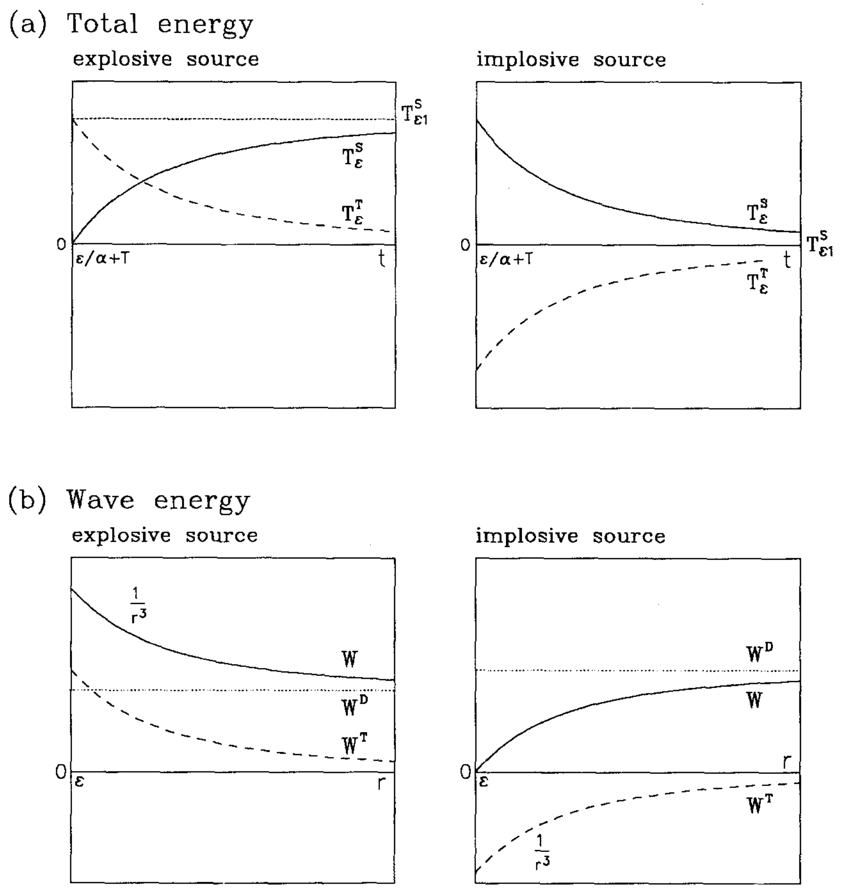

Figure 6

(a) Total transient energy T_ε^T (dashed line) and total static energy T_ε^S (solid line) as a function of time for the isotropic explosive and inner implosive source. The functions are plotted for times after the source process has ended ($t \geq \varepsilon/\alpha + T$). $T_{\varepsilon 1}^S$ denotes the limit of $T_\varepsilon^S(t)$ as $t \to \infty$. (b) Wave energy W (solid line), transient wave energy W^T (dashed line) and dynamic wave energy W^D (dotted line) as a function of distance. Functions are plotted for distances greater than source radius ε.

source process has ended. Due to the release of static energy, this energy deficit decreases and vanishes when all the static energy has been released (Fig. 6a). For the *inner implosive source*, wave energy $W^T(r)$ is a negative and increasing function of distance (Fig. 6b). The increase of $W^T(r)$ is due to the transformation of static into transient wave energy. Transient energy flux $P^T(r)$ (15c) is directed towards the source which results in the waves transporting a part of energy in a direction opposite to the wave propagation. $W^T(r)$ and $P^T(r)$ vanish once the static energy has been released.

Residual energy H^R (12e) is also local (Fig. 5b), consisting of potential as well as kinetic energies. It has a zero total value (13c, 16c), implying that it does not contribute to the total energy balance and redistributes energy only among various elastic waves. It is noted, however, that due to its relatively slow decrease with distance ($\propto 1/r$), H^R still considerably affects the time-space form of the surface energy $H(r, t)$ at the distances, at which H^S and H^T are practically zero. Figures 7a,b indicate that irregularities Δ in $H(r, t)$, introduced by H^R, can comprise 10% of the peak amplitude of H at distances greater than 5 wavelengths from the source.

5. Double-couple Source

5.1. Displacement Field

Next, we assume a double-couple source at the origin of coordinates with the moment tensor in the form

$$M_{ij}(t) = (\delta_{1i}\delta_{2j} + \delta_{1j}\delta_{2i})M(t).$$

Displacement field $u_i(\vec{x}, t)$ then is expressed (AKI and RICHARDS, 1980) by:

$$\begin{aligned} u_i(\vec{x}, t) = {} & \frac{1}{4\pi\rho}(30\gamma_i\gamma_1\gamma_2 - 6\gamma_1\delta_{i2} - 6\gamma_2\delta_{i1})\frac{1}{r^4}\int_{r/\alpha}^{r/\beta} \tau M(t-\tau)\,d\tau \\ & + \frac{1}{4\pi\rho\alpha^2}(12\gamma_i\gamma_1\gamma_2 - 2\gamma_1\delta_{i2} - 2\gamma_2\delta_{i1})\frac{M(t-r/\alpha)}{r^2} \\ & - \frac{1}{4\pi\rho\beta^2}(12\gamma_i\gamma_1\gamma_2 - 3\gamma_1\delta_{i2} - 3\gamma_2\delta_{i1})\frac{M(t-r/\beta)}{r^2} + \frac{2\gamma_i\gamma_1\gamma_2}{4\pi\rho\alpha^3}\frac{\dot{M}(t-r/\alpha)}{r} \\ & - \frac{1}{4\pi\rho\beta^3}(2\gamma_i\gamma_1\gamma_2 - \gamma_1\delta_{i2} - \gamma_2\delta_{i1})\frac{\dot{M}(t-r/\beta)}{r}. \end{aligned} \tag{17}$$

The first three terms in (17) are called *near-field waves*, and we will denote them I^N, P^N and S^N *waves*, respectively. The last two terms in (17) represent *far-field P waves* (P^F) and *far-field S waves* (S^F). The amplitude of far-field waves decreases as $1/r$, and $1/r^2$ or faster for near-field waves. The P^N and P^F waves propagate with

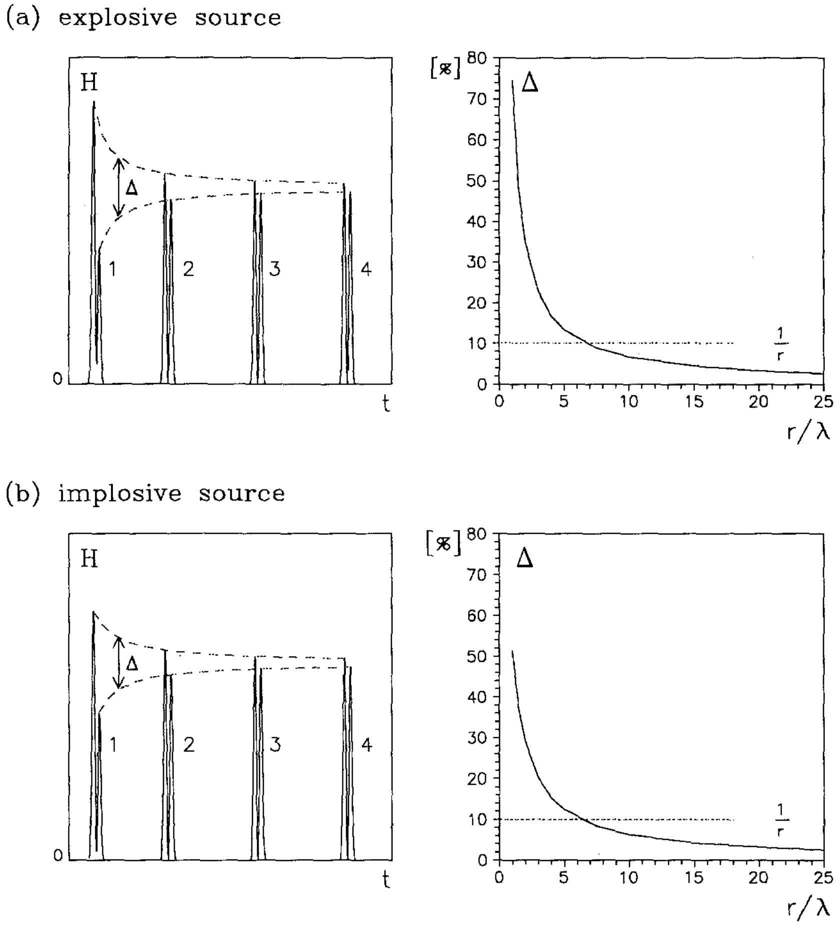

Figure 7

Surface energy $H(r, t)$ for the isotropic source as a function of time (left) and difference Δ between peak amplitudes of H as a function of r/λ (right). $\Delta = 2(\alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2) \cdot 100\%$, where $\alpha_1(\alpha_2)$ is the first (second) peak amplitude of H. The time function of H is shown at distances: (1) $r/\lambda^P = 1$, (2) $r/\lambda^P = 5$, (3) $r/\lambda^P = 10$, (4) $r/\lambda^P = 15$. Other parameters used: $\alpha = 5.6$ km/s, $T = 0.5$ s, $\lambda^P = 2.8$ km. The dotted line denotes the 10% level of Δ, which can be considered as the lower limit of the significant contamination of the wavefield by near-field waves.

velocity α, the S^N and S^F waves with β. The I^N waves lack a single propagation velocity but are intrinsically dispersed. The P^F and S^F waves are linearly polarized, P^F waves being purely longitudinal and S^F waves purely transverse. The I^N, P^N and S^N waves are also polarized linearly, but their polarization is oblique to their propagation direction. The radiation patterns of P^F and S^F waves have the well-known quatrefoil form, while the polarization and radiation patterns of the total wavefield are more complicated. See VAVRYČUK (1992) for details.

Source-time function $M(t)$ will be chosen identically for the isotropic source (Fig. 1). Instead of explosive and implosive sources, we will address dislocations and antidislocations (ANDREWS, 1975). By the *dislocation source* we understand a double-couple source effecting a transition from an unstressed to a stressed state of the medium, and *vice versa* for the *antidislocation source.* By the *inner* antidislocation we will understand a spontaneous antidislocation, i.e., an antidislocation which forms an isolated system with the medium.

5.2. Total and Wave Energy

The formulae for energy density $h(\vec{x}, t)$, for energy flux density $\vec{p}(\vec{x}, t)$, surface energies $H_K(r, t)$ and $H_U(r, t)$, and energy flux $P(r, t)$ can be established analogously to those for the isotropic source. However, the double-couple derivation is rather long and the resulting formulae are exceedingly more complicated. Some complete forms are given in the Appendix. In this section only the final analytical formulae for total energy $T_\varepsilon(t)$ and wave energy $W(r)$ will be discussed.

For $t \leq \varepsilon/\alpha$, (4a) and (5b) together with (A1) and (A2) yield

$$T_{\varepsilon 0} = T_\varepsilon(t \leq \varepsilon/\alpha) = \frac{K}{60\pi\rho^2}\frac{M_0^2}{\varepsilon^3}, \tag{18}$$

where

$$K = \frac{6\lambda}{\alpha^4} + \frac{32\mu}{\alpha^4} + \frac{27\mu}{\beta^4} - \frac{48\mu}{\alpha^2\beta^2}. \tag{19}$$

After the source process has ended, $t \geq \varepsilon/\beta + T$,

$$T_{\varepsilon 1} = T_\varepsilon(t \geq \varepsilon/\beta + T) = \frac{1}{60\pi\rho^2}\left\{\frac{KM_1^2}{\varepsilon^3} + \frac{4(\lambda + 2\mu)}{\alpha^7}\int_0^T \ddot{M}^2(t)\,dt + \frac{6\mu}{\beta^7}\int_0^T \ddot{M}^2(t)\,dt\right\}. \tag{20}$$

For the *dislocation source,* the first term in (20) expresses the static potential energy, which is loaded into the medium by external forces during the source process. The second and the third terms denote the energy of P and S far-field waves, respectively, radiated outwards from the source into infinity. For the *antidislocation source*, the first term in (20) equals zero, consequently the final total energy equals the energy of P and S far-field waves only.

For wave energy $W(r)$ formulae (A3) and (5c) yield

$$W(r) = \frac{1}{60\pi\rho^2}\left\{\frac{K(M_1^2 - M_0^2)}{r^3} + \frac{4(\lambda + 2\mu)}{\alpha^7}\int_0^T \ddot{M}^2(t)\,dt + \frac{6\mu}{\beta^7}\int_0^T \ddot{M}^2(t)\,dt\right\}. \tag{21}$$

The first term in (21) expresses the near-field wave energy, while the second and third terms express the P and S far-field wave energies, respectively.

5.3. Types of Energy and Energy Flux: Formulae

In analogy with the isotropic source, we can split the elastic energy into a sum of static, dynamic, transient and residual energies:

$$H = H^S + H^D + H^T + H^R, \tag{22a}$$

$$H^S = \frac{1}{60\pi\rho^2}\left\{\frac{18\lambda + 96\mu}{\alpha^4}\frac{M_\alpha^2}{r^4} + \frac{81\mu}{\beta^4}\frac{M_\beta^2}{r^4} - \frac{144\mu}{\alpha^2\beta^2}\frac{M_\alpha M_\beta}{r^4}\right\}, \tag{22b}$$

$$H^D = \frac{1}{60\pi\rho^2}\left\{\frac{4(\lambda + 2\mu)}{\alpha^8}\ddot{M}_\alpha^2 + \frac{6\mu}{\beta^8}\ddot{M}_\beta^2\right\}, \tag{22c}$$

$$H^T = \frac{1}{60\pi\rho^2}\left\{\frac{12\lambda + 64\mu}{\alpha^5}\frac{M_\alpha\dot{M}_\alpha}{r^3} + \frac{54\mu}{\beta^5}\frac{M_\beta\dot{M}_\beta}{r^3} - \frac{48\mu}{\alpha^3\beta^2}\frac{\dot{M}_\alpha M_\beta}{r^3} - \frac{48\mu}{\alpha^2\beta^3}\frac{M_\alpha\dot{M}_\beta}{r^3}\right\}, \tag{22d}$$

where $M_\alpha = M(t - r/\alpha)$ and $M_\beta = M(t - r/\beta)$. Formula for H^R is very complex and omitted here. Integrating H^S, H^D and H^R over r, we get

$$T_{\varepsilon 0}^S = \frac{KM_0^2}{60\pi\rho^2}\frac{1}{\varepsilon^3}, \quad T_{\varepsilon 1}^S = \frac{KM_1^2}{60\pi\rho^2}\frac{1}{\varepsilon^3}, \tag{23a}$$

$$T_\varepsilon^D(t \geq \varepsilon/\beta + T) = \frac{1}{60\pi\rho^2}\left\{\frac{4(\lambda + 2\mu)}{\alpha^7}\int_0^T \ddot{M}^2(t)\,dt + \frac{6\mu}{\beta^7}\int_0^T \ddot{M}^2(t)\,dt\right\}, \tag{23b}$$

$$T_\varepsilon^R(t \geq \varepsilon/\beta + T) = 0. \tag{23c}$$

Integrating (22b,d) for $t \geq \varepsilon/\beta + T$, we get

$$T_\varepsilon^S + T_\varepsilon^T = T_{\varepsilon 1}^S. \tag{24}$$

Analogously to (15) we will define the dynamic, transient and residual parts of energy flux P as follows:

$$P = P^D + P^T + P^R, \tag{25a}$$

$$P^D = \frac{1}{60\pi\rho^2}\left\{\frac{4(\lambda + 2\mu)}{\alpha^7}\ddot{M}_\alpha^2 + \frac{6\mu}{\beta^7}\ddot{M}_\beta^2\right\}, \tag{25b}$$

$$P^T = \frac{1}{60\pi\rho^2}\left\{\frac{12\lambda + 64\mu}{\alpha^4}\frac{M_\alpha\dot{M}_\alpha}{r^3} + \frac{54\mu}{\beta^4}\frac{M_\beta\dot{M}_\beta}{r^3} - \frac{48\mu}{\alpha^2\beta^2}\frac{\dot{M}_\alpha M_\beta}{r^3} - \frac{48\mu}{\alpha^2\beta^2}\frac{M_\alpha\dot{M}_\beta}{r^3}\right\}. \tag{25c}$$

P^R can be derived from (A3) and (25) but not presented here. Integrating P^D, P^T and P^R over t, we get

$$W^D = \frac{1}{60\pi\rho^2}\left\{\frac{4(\lambda + 2\mu)}{\alpha^7}\int_0^T \ddot{M}^2(t)\,dt + \frac{6\mu}{\beta^7}\int_0^T \ddot{M}^2(t)\,dt\right\}, \tag{26a}$$

$$W^T = \frac{K}{60\pi\rho^2}\frac{M_1^2 - M_0^2}{r^3}, \tag{26b}$$

$$W^R = 0. \tag{26c}$$

5.4. *Types of Energy and Energy Flux: Discussion*

Formula (22b) indicates that the *static energy* H^S consists of the energy related to P^N and S^N waves and the energy related to the interaction between them. The terms for the interactions of P^N–P^N and S^N–S^N are always positive, while the P^N–S^N interaction term is always negative (see Fig. 8a). Any change of static energy can be effected in time-space intervals $r/\alpha \leq t \leq r/\alpha + T$ or $r/\beta \leq t \leq r/\beta + T$.

The *dynamic energy* H^D (22c) splits into two terms: the dynamic energies transported by P^F waves and S^F waves. Both terms are always positive (Fig. 8b). The dynamic term for the interaction of P^F–S^F is zero.

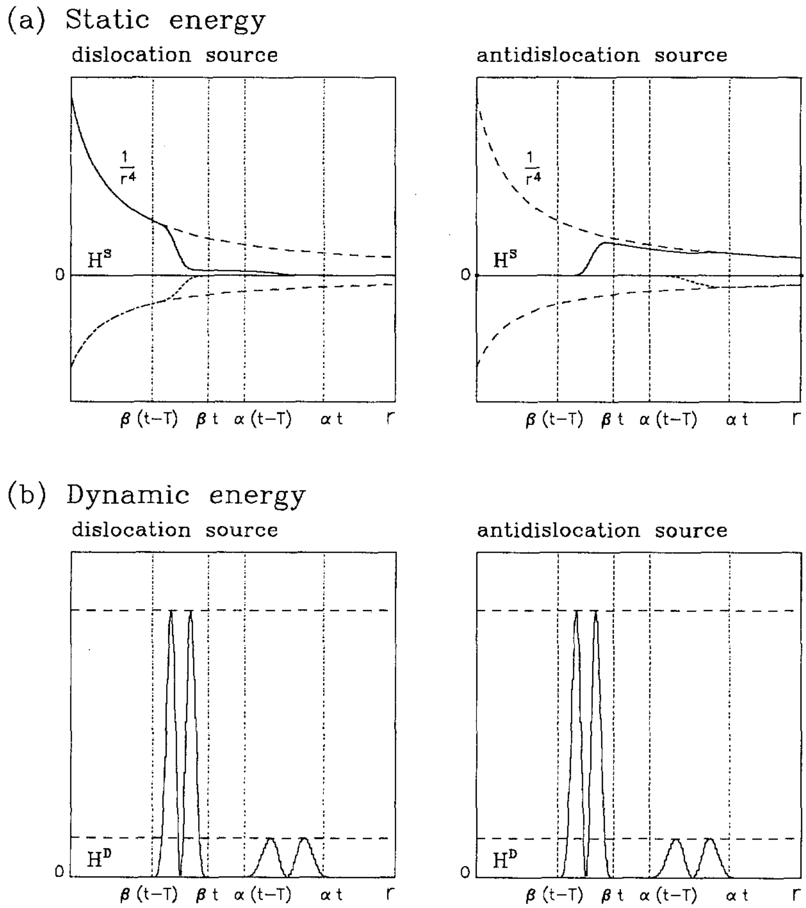

Figure 8

(a) Schematic plot of static surface energy H^S for the double-couple source as a function of distance at a fixed instant t. H^S is divided into two parts: P–S interaction term (dotted line) and the sum of $P-P$ and S–S terms (solid line). The vertical dotted lines delineate the space interval of the P and S wave. The dashed line shows the envelope of H^S for different times. (b) Dynamic surface energy H^D as a function of distance at a fixed instant t.

The *transient energy* H^T (22d) is local energy, either positive or negative (Fig. 9a). Comparing (14) with (24), we conclude that the static-to-transient energy transformation for the double-couple is quite analogous to that for the isotropic sources. However, not only the P-wave energy but also the S-wave energy or the interaction energy of $P-S$ exist in the energy balance.

As for the isotropic source the *residual energy* H^R (Fig. 9b) does not contribute to the total energy balance (23c, 26c) and, therefore, it is responsible only for the redistribution of the energy among elastic waves. However, Figures 10a,b show that irregularities Δ in $H(r, t)$, introduced by H^R, are considerably more pronounced than for the isotropic source, nearly 10% of peak amplitude of H at distances greater than 20 wavelengths from the source.

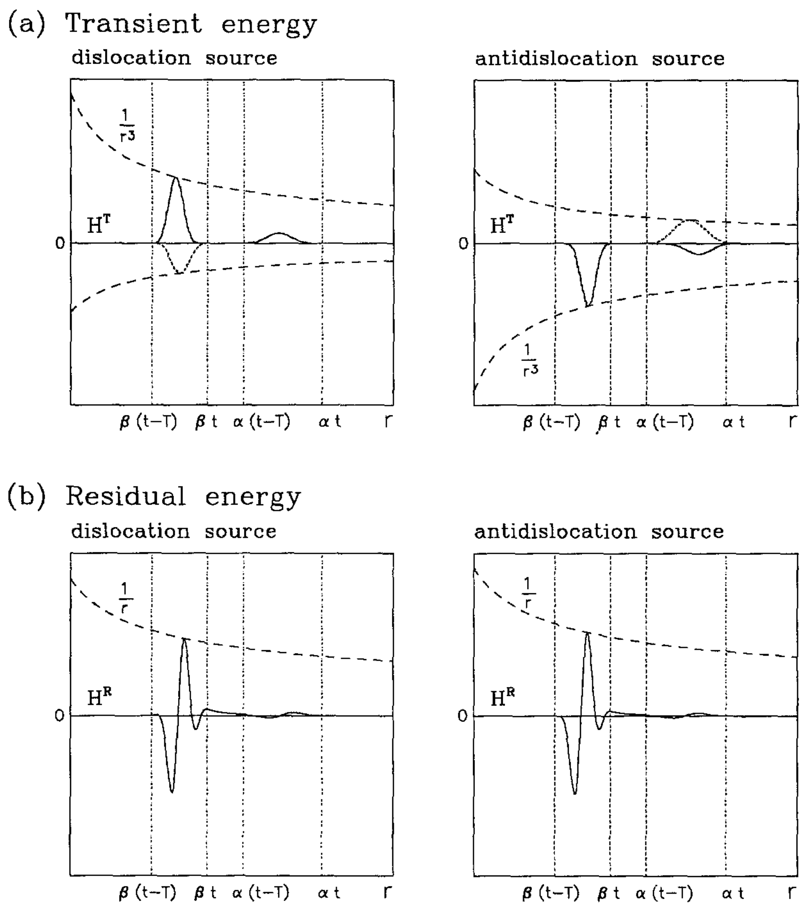

Figure 9
Schematic plot of transient surface energy H^T (a) and residual surface energy H^R (b) for the double-couple source as a function of distance at a fixed instant t.

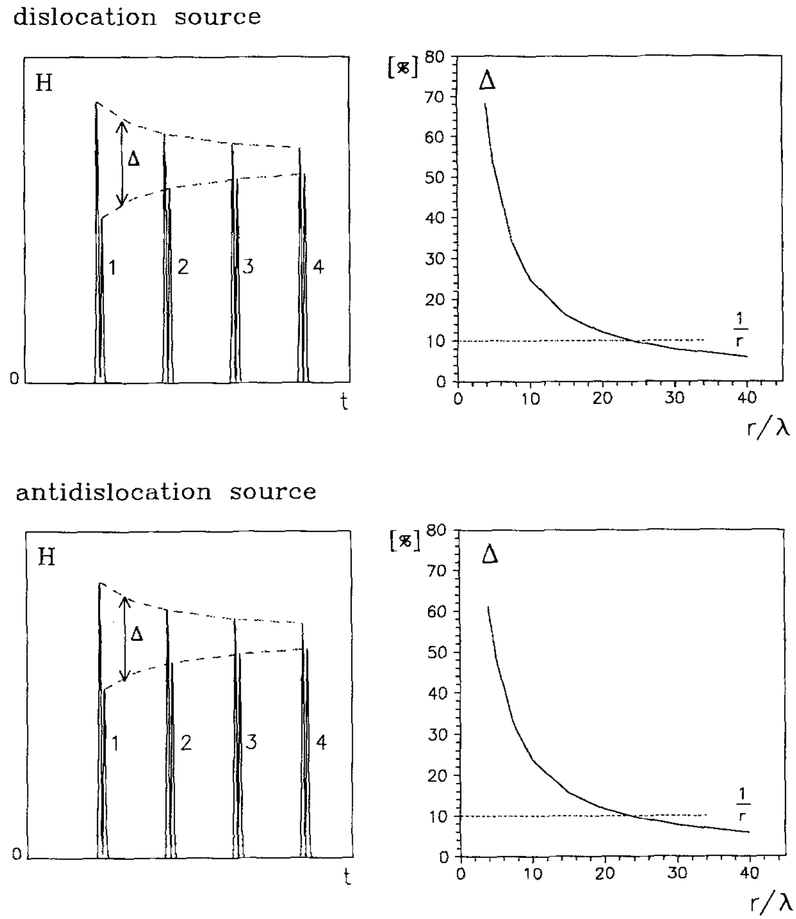

Figure 10
Surface energy $H(r, t)$ as a function of time (left) and difference Δ between peak amplitudes of H as a function of r/λ (right) for the double-couple source. The time function of H is shown at distances: (1) $r/\lambda^P = 5$, (2) $r/\lambda^P = 10$, (3) $r/\lambda^P = 15$, (4) $r/\lambda^P = 20$. Other parameters used: $\alpha = 5.6$ km/s, $\beta = \alpha/\sqrt{3}$, $T = 0.5$ s, $\lambda^P = 2.8$ km. For other details see the caption of Fig. 7.

6. *Conclusion*

The elastic energy radiated by the isotropic as well as by the double-couple source consists of several physically different energy forms: static, dynamic, transient and residual energies. The *static energy* is a nonlocal potential energy related to static deformations. In an equilibrium state it is distributed throughout the medium, being mainly concentrated around a source. Static surface energy H^S is always positive and decreases with the fourth power of distance from the source. The *dynamic energy* is a local one transported by far-field waves and consisting equally of potential and kinetic energies. This energy is always positive, and its total amount contained in the elastic waves remains constant regardless of distance and time. The dynamic energy corresponds to a usually determined *radiated energy*. The *transient energy* is a local potential energy transported by the near-field waves. It can be either positive or negative, and it is responsible for loading or releasing the nonlocal static energy. The *residual energy* is local, a sum of potential and kinetic energies, originating in the mutual interaction of the near-field and the far-field waves. Residual surface energy H^R decreases as $1/r$, affecting the time-space form of elastic surface energy $H(r, t)$ at relatively large distances from the source. The kinetic as well as potential parts of total energy T^R and wave energy W^R equal zero. Since the static and transient energies are potential energies, and the kinetic part of the residual wave energy equals zero, we can draw an interesting conclusion that the kinetic part of the wave energy W equals *exactly* a kinetic part of the dynamic wave energy W^D. Kinetic wave energy of the complete wavefield does not therefore depend on the distance from the source and equals exactly the kinetic wave energy of the far-field waves. It opens a possibility, of determining *exactly* the far-field wave energy, which equals double their kinetic energy, from the wavefield measured at any distance from the source, including its close vicinity.

For the *explosive isotropic source*, the work carried out by the external forces at the source surface is stored in waves immediately after the source process termination. Wave energy has its maximum at the source surface, and it decreases with increasing distance, converging to the far-field wave energy. The decrease of the wave energy is due to the transformation of the wave-to-static energy. Loading of the static energy into the medium is gradual, in accordance with the causality principle, performed by the near-field waves. For the *implosive isotropic source*, all the energy is stored in the form of static energy prior to the source process. This energy is not concentrated at the source but in the medium surrounding it. During the implosion the source surface generates elastic waves without requiring additional radiation of positive wave energy. For an *inner source*, the source surface generates waves, but radiates no wave energy. Waves propagating from the source release the static energy stored in the medium and transform it into the wave energy. The wave energy increases, converging to the far-field wave energy in infinity.

The formulae for the elastic energy and the energy flux of waves generated by the *double-couple source* have a substantially more complicated form. The radiation patterns of the elastic energy density and the energy flux density are not spherically symmetrical. The energy flux density is not radial, with nonzero values not only at times of the P waves ($r/\alpha \leq t \leq r/\alpha + T$) and of the S waves ($r/\beta \leq t \leq r/\beta + T$), but also at times between them ($r/\alpha + T \leq t \leq r/\beta$). The static and transient energies consist of three terms: the energies related to P waves, to S waves, and to their mutual interaction. The dynamic energy is divided into the energies transmitted by the P and S waves. Their interaction term is identically zero. Although more energy terms are involved in energy balance compared to the isotropic source, general principles of energy balance remain the same. For the *dislocation source*, dynamic energy is radiated by the far-field P and S waves outwards from the source into infinity; transient energy is transformed into the static energy by the near-field waves. For the *inner antidislocation*, wave energy is zero at the source surface, increasing with distance due to the static-to-wave energy transformation. The transient wave energy is negative and the transient energy flux is directed towards the source, in the direction opposite to the wave propagation, vanishing at large distances. Simply, for *explosive* and *dislocation sources*, *the near-field waves transport and distribute the static energy into the medium*, with the result that the wave energy decreases. In contrast, for *implosive* and *antidislocation sources*, *the near-field waves propagating through a stressed medium release the static energy from the medium*, transforming it into wave energy, so that it increases.

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Appendix

Analytical Formulae for Energy and Energy Flux: Double-couple Source

Formulae for $H_K(r, t)$, $H_U(r, t)$ and $P(r, t)$ can be established by substituting (17) to (1), (2), (4a) and (4b)

$$
\begin{aligned}
H_K(r, t) = \frac{\lambda + 2\mu}{60\pi\rho^2} \Bigg\{ & \frac{270}{\alpha^2}\frac{I_1^2}{r^6} + \frac{540}{\alpha^3}\frac{M_\alpha I_1}{r^5} - \frac{540}{\alpha^2\beta}\frac{M_\beta I_1}{r^5} + \frac{216}{\alpha^4}\frac{\dot{M}_\alpha I_1}{r^4} - \frac{216}{\alpha^2\beta^2}\frac{\dot{M}_\beta I_1}{r^4} \\
& + \frac{270}{\alpha^4}\frac{M_\alpha^2}{r^4} + \frac{270}{\alpha^2\beta^2}\frac{M_\beta^2}{r^4} - \frac{540}{\alpha^3\beta}\frac{M_\alpha M_\beta}{r^4} + \frac{36}{\alpha^5}\frac{\ddot{M}_\alpha I_1}{r^3} - \frac{36}{\alpha^2\beta^3}\frac{\ddot{M}_\beta I_1}{r^3} \\
& + \frac{216}{\alpha^5}\frac{M_\alpha \dot{M}_\alpha}{r^3} + \frac{216}{\alpha^2\beta^3}\frac{M_\beta \dot{M}_\beta}{r^3} - \frac{216}{\alpha^3\beta^2}\frac{M_\alpha \dot{M}_\beta}{r^3} - \frac{216}{\alpha^4\beta}\frac{\dot{M}_\alpha M_\beta}{r^3} + \frac{36}{\alpha^6}\frac{M_\alpha \ddot{M}_\alpha}{r^2} \\
& + \frac{36}{\alpha^2\beta^4}\frac{M_\beta \ddot{M}_\beta}{r^2} - \frac{36}{\alpha^3\beta^3}\frac{M_\alpha \ddot{M}_\beta}{r^2} - \frac{36}{\alpha^5\beta}\frac{\ddot{M}_\alpha M_\beta}{r^2} + \frac{44}{\alpha^6}\frac{\dot{M}_\alpha^2}{r^2} + \frac{45}{\alpha^2\beta^4}\frac{\dot{M}_\beta^2}{r^2} \\
& - \frac{84}{\alpha^4\beta^2}\frac{\dot{M}_\alpha \dot{M}_\beta}{r^2} + \frac{16}{\alpha^7}\frac{\dot{M}_\alpha \ddot{M}_\alpha}{r} + \frac{18}{\alpha^2\beta^5}\frac{\dot{M}_\beta \ddot{M}_\beta}{r} - \frac{12}{\alpha^4\beta^3}\frac{\dot{M}_\alpha \ddot{M}_\beta}{r} \\
& - \frac{12}{\alpha^5\beta^2}\frac{\ddot{M}_\alpha \dot{M}_\beta}{r} + \frac{2}{\alpha^8}\ddot{M}_\alpha^2 + \frac{3}{\alpha^2\beta^6}\ddot{M}_\beta^2 \Bigg\},
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
H_U(r,t) = {} & \frac{\lambda}{60\pi\rho^2}\left\{\frac{18}{\alpha^4}\frac{M_\alpha^2}{r^4}+\frac{36}{\alpha^5}\frac{M_\alpha\dot{M}_\alpha}{r^3}+\frac{12}{\alpha^6}\frac{M_\alpha\ddot{M}_\alpha}{r^2}\right. \\
& \left.+\frac{18}{\alpha^6}\frac{\dot{M}_\alpha^2}{r^2}+\frac{12}{\alpha^7}\frac{\dot{M}_\alpha\ddot{M}_\alpha}{r}+\frac{2}{\alpha^8}\ddot{M}_\alpha^2\right\} \\
& +\frac{\mu}{60\pi\rho^2}\left\{15120\frac{I_0^2}{r^8}+\frac{12960}{\alpha^2}\frac{M_\alpha I_0}{r^6}-\frac{12960}{\beta^2}\frac{M_\beta I_0}{r^6}+\frac{2880}{\alpha^3}\frac{\dot{M}_\alpha I_0}{r^5}\right. \\
& -\frac{2880}{\beta^3}\frac{\dot{M}_\beta I_0}{r^5}+\frac{288}{\alpha^4}\frac{\ddot{M}_\alpha I_0}{r^4}-\frac{288}{\beta^4}\frac{\ddot{M}_\beta I_0}{r^4}+\frac{2796}{\alpha^4}\frac{M_\alpha^2}{r^4}+\frac{2781}{\beta^4}\frac{M_\beta^2}{r^4} \\
& -\frac{5544}{\alpha^2\beta^2}\frac{M_\alpha M_\beta}{r^4}+\frac{1272}{\alpha^5}\frac{M_\alpha\dot{M}_\alpha}{r^3}+\frac{1242}{\beta^5}\frac{M_\beta\dot{M}_\beta}{r^3}-\frac{1224}{\alpha^2\beta^3}\frac{M_\alpha\dot{M}_\beta}{r^3} \\
& -\frac{1224}{\alpha^3\beta^2}\frac{\dot{M}_\alpha M_\beta}{r^3}+\frac{136}{\alpha^6}\frac{M_\alpha\ddot{M}_\alpha}{r^2}+\frac{126}{\beta^6}\frac{M_\beta\ddot{M}_\beta}{r^2}-\frac{120}{\alpha^2\beta^4}\frac{M_\alpha\ddot{M}_\beta}{r^2} \\
& -\frac{120}{\alpha^4\beta^2}\frac{\ddot{M}_\alpha M_\beta}{r^2}+\frac{156}{\alpha^6}\frac{\dot{M}_\alpha^2}{r^2}+\frac{141}{\beta^6}\frac{\dot{M}_\beta^2}{r^2}-\frac{264}{\alpha^3\beta^3}\frac{\dot{M}_\alpha\dot{M}_\beta}{r^2}+\frac{40}{\alpha^7}\frac{\dot{M}_\alpha\ddot{M}_\alpha}{r} \\
& \left.+\frac{30}{\beta^7}\frac{\dot{M}_\beta\ddot{M}_\beta}{r}-\frac{24}{\alpha^3\beta^4}\frac{\dot{M}_\alpha\ddot{M}_\beta}{r}-\frac{24}{\alpha^4\beta^3}\frac{\ddot{M}_\alpha\dot{M}_\beta}{r}+\frac{4}{\alpha^8}\ddot{M}_\alpha^2+\frac{3}{\beta^8}\ddot{M}_\beta^2\right\},
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
P(r,t) = {} & \frac{\lambda}{60\pi\rho^2}\left\{\frac{108}{\alpha^2}\frac{M_\alpha I_1}{r^5}+\frac{108}{\alpha^3}\frac{\dot{M}_\alpha I_1}{r^4}+\frac{108}{\alpha^3}\frac{M_\alpha^2}{r^4}-\frac{108}{\alpha^2\beta}\frac{M_\alpha M_\beta}{r^4}+\frac{36}{\alpha^4}\frac{\ddot{M}_\alpha I_1}{r^3}\right. \\
& +\frac{156}{\alpha^4}\frac{M_\alpha\dot{M}_\alpha}{r^3}-\frac{108}{\alpha^3\beta}\frac{\dot{M}_\alpha M_\beta}{r^3}-\frac{36}{\alpha^2\beta^2}\frac{M_\alpha\dot{M}_\beta}{r^3}+\frac{48}{\alpha^5}\frac{M_\alpha\ddot{M}_\alpha}{r^2}-\frac{36}{\alpha^4\beta}\frac{\ddot{M}_\alpha M_\beta}{r^2} \\
& \left.+\frac{48}{\alpha^5}\frac{\dot{M}_\alpha^2}{r^2}-\frac{36}{\alpha^3\beta^2}\frac{\dot{M}_\alpha\dot{M}_\beta}{r^2}+\frac{28}{\alpha^6}\frac{\dot{M}_\alpha\ddot{M}_\alpha}{r}-\frac{12}{\alpha^4\beta^2}\frac{\ddot{M}_\alpha\dot{M}_\beta}{r}+\frac{4}{\alpha^7}\ddot{M}_\alpha^2\right\} \\
& +\frac{\mu}{60\pi\rho^2}\left\{4320\frac{I_1 I_0}{r^7}+\frac{4320}{\alpha}\frac{M_\alpha I_0}{r^6}-\frac{4320}{\beta}\frac{M_\beta I_0}{r^6}+\frac{1728}{\alpha^2}\frac{\dot{M}_\alpha I_0}{r^5}\right. \\
& -\frac{1728}{\beta^2}\frac{\dot{M}_\beta I_0}{r^5}+\frac{1944}{\alpha^2}\frac{M_\alpha I_1}{r^5}-\frac{1836}{\beta^2}\frac{M_\beta I_1}{r^5}+\frac{288}{\alpha^3}\frac{\ddot{M}_\alpha I_0}{r^4}-\frac{288}{\beta^3}\frac{\ddot{M}_\beta I_0}{r^4} \\
& +\frac{504}{\alpha^3}\frac{\dot{M}_\alpha I_1}{r^4}-\frac{396}{\beta^3}\frac{\dot{M}_\beta I_1}{r^4}+\frac{1944}{\alpha^3}\frac{M_\alpha^2}{r^4}+\frac{1836}{\beta^3}\frac{M_\beta^2}{r^4}-\frac{1836}{\alpha\beta^2}\frac{M_\alpha M_\beta}{r^4} \\
& -\frac{1944}{\alpha^2\beta}\frac{M_\alpha M_\beta}{r^4}+\frac{72}{\alpha^4}\frac{\ddot{M}_\alpha I_1}{r^3}-\frac{36}{\beta^4}\frac{\ddot{M}_\beta I_1}{r^3}+\frac{1288}{\alpha^4}\frac{M_\alpha\dot{M}_\alpha}{r^3}+\frac{1134}{\beta^4}\frac{M_\beta\dot{M}_\beta}{r^3} \\
& -\frac{768}{\alpha^2\beta^2}\frac{M_\alpha\dot{M}_\beta}{r^3}-\frac{396}{\alpha\beta^3}\frac{M_\alpha\dot{M}_\beta}{r^3}-\frac{732}{\alpha^2\beta^2}\frac{\dot{M}_\alpha M_\beta}{r^3}-\frac{504}{\alpha^3\beta}\frac{\dot{M}_\alpha M_\beta}{r^3} \\
& +\frac{208}{\alpha^5}\frac{M_\alpha\ddot{M}_\alpha}{r^2}+\frac{162}{\beta^5}\frac{M_\beta\ddot{M}_\beta}{r^2}-\frac{120}{\alpha^2\beta^3}\frac{M_\alpha\ddot{M}_\beta}{r^2}-\frac{36}{\alpha\beta^4}\frac{M_\alpha\ddot{M}_\beta}{r^2}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{120}{\alpha^3\beta^2}\frac{\ddot{M}_\alpha M_\beta}{r^2}-\frac{72}{\alpha^4\beta}\frac{\ddot{M}_\alpha M_\beta}{r^2}+\frac{208}{\alpha^5}\frac{\dot{M}_\alpha^2}{r^2}+\frac{162}{\beta^5}\frac{\dot{M}_\beta^2}{r^2}-\frac{192}{\alpha^3\beta^2}\frac{\dot{M}_\alpha\dot{M}_\beta}{r^2}\\
&-\frac{156}{\alpha^2\beta^3}\frac{\dot{M}_\alpha\dot{M}_\beta}{r^2}+\frac{72}{\alpha^6}\frac{\dot{M}_\alpha\ddot{M}_\alpha}{r}+\frac{48}{\beta^6}\frac{\dot{M}_\beta\ddot{M}_\beta}{r}-\frac{24}{\alpha^3\beta^3}\frac{\dot{M}_\alpha\ddot{M}_\beta}{r}\\
&-\frac{12}{\alpha^2\beta^4}\frac{\dot{M}_\alpha\ddot{M}_\beta}{r}-\frac{24}{\alpha^3\beta^3}\frac{\ddot{M}_\alpha\dot{M}_\beta}{r}-\frac{24}{\alpha^4\beta^2}\frac{\ddot{M}_\alpha\dot{M}_\beta}{r}+\frac{8}{\alpha^7}\ddot{M}_\alpha^2+\frac{6}{\beta^7}\ddot{M}_\beta^2\Bigg\},
\end{aligned}
\tag{A3}
$$

where

$$
I_0=\int_{r/\alpha}^{r/\beta}\tau M(t-\tau)\,d\tau,\quad I_1=\int_{r/\alpha}^{r/\beta}M(t-\tau)\,d\tau,
$$

$$
M_\alpha=M(t-r/\alpha)\quad\text{and}\quad M_\beta=M(t-r/\beta).
$$

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