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Weak Contrast *PP* Wave Displacement R/T Coefficients in Weakly Anisotropic Elastic Media

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Abstract—Approximate *PP* plane wave displacement coefficients of reflection and transmission for weak contrast interfaces separating weakly but arbitrarily anisotropic elastic media are presented. The *PP* reflection coefficient for such an interface has been derived recently by VAVRYČUK and PŠENČÍK (1997). The *PP* transmission coefficient presented in this paper was derived by the same approach. The coefficients are given as a sum of the coefficient for the weak contrast interface separating two nearby isotropic media and a term depending linearly on contrasts of the so-called weak anisotropy (WA) parameters (parameters specifying deviation of properties of the medium from isotropy), across the interface. While the reflection coefficient depends only on 8 of the complete set of the WA parameters describing *P*-wave phase velocity in weakly anisotropic media, the transmission coefficient depends on their complete set. The *PP* reflection coefficient depends on "shear-wave splitting parameter" γ. Tests of accuracy of the approximate formulae are presented on several models.

Key words: Weak anisotropy, weak contrast interface, plane wave reflection and transmission coefficients.

1. Introduction

Seismic anisotropy is a nearly omnipresent phenomenon, which affects, often considerably, parameters of propagating elastic waves. Displacement coefficients of reflection and transmission (R/T) belong to such parameters. For isotropic media, explicit formulae for the R/T coefficients are well known, see e.g., AKI and RICHARDS (1980). They are, however, relatively complicated and their relation to elastic parameters is often strongly nonlinear. The complexity of the coefficients reduces substantially if the contrast between the two media separated by an interface is weak. The coefficients can be linearized with respect to the contrast in elastic parameters. The linearized formulae become more transparent and they are often very accurate, see again AKI and RICHARDS (1980). In anisotropic media, explicit expressions for the R/T coefficients are available for media with a higher symmetry, whose symmetry planes are especially oriented with respect to an interface, see e.g. DALEY and HRON (1977), KEITH and CRAMPIN (1977). In the

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case of general anisotropy, a common method to determine the coefficients is the numerical solution of the system of equations resulting from the boundary conditions, see e.g. GAJEWSKI and PŠENČÍK (1987). As in isotropic media, the problem simplifies considerably if reflection/transmission at a weak contrast interface is considered and if the media surrounding the interface are only weakly anisotropic.

The assumption of weak contrast interface and of weak anisotropy has been used by several authors although higher symmetry anisotropy was always considered. THOMSEN (1993) extended BANIK's (1987) work and derived the *PP* R/T coefficients for a weak contrast interface separating two weakly transversely isotropic media with axes of symmetry perpendicular to the interface, see also discussion of this formula by TSVANKIN (1996). RUEGER (1996) corrected and generalized Thomsen's results for *PP* reflections in planes containing symmetry axes of transversely isotropic and orthorhombic media so that media with symmetry axes parallel to the interface could also be considered. HAUGEN and URSIN (1996) derived *PP* reflection coefficients in the symmetry planes of a model containing an interface separating a TI medium with axis of symmetry perpendicular to the interface from a TI medium with axis of symmetry parallel to the interface.

This paper is an extension of the paper by VAVRYČUK and PŠENČÍK (1998) who derived an approximate formula for the *PP* wave displacement coefficient of reflection for a weak contrast interface separating two arbitrary weakly anisotropic media. In this paper, in addition to the formula for the reflection coefficient, the *PP* wave displacement coefficient of transmission is presented. Both formulae are obtained by applying the first-order perturbation theory. Continuous isotropic medium with no discontinuity of parameters of the medium across the studied interface is considered as a background medium. The media on both sides of the interface are then perturbed so that the result is a model composed of two slightly different weakly anisotropic halfspaces.

Accuracy of the approximate formulae is tested on models consisting of a homogeneous isotropic halfspace over a halfspace filled by a homogeneous transversely isotropic (TI) material with the horizontal axis of symmetry (the HTI material). Behavior of the *PP* R/T coefficients at an interface separating two TI halfspaces is also shown. The upper halfspace contains a material with the vertical axis of symmetry (the VTI material), the lower halfspace contains the HTI material or a material with the inclined axis of symmetry (the ITI material).

If not specified differently, the Roman lower-case indices attain values 1, 2 and 3, upper-case Roman indices attain only values 1 and 2. The Greek indices run from 1 to 6. Einstein summation convention is used for the repeated indices.

2. Basic Formulae

VAVRYČUK and PŠENČÍK (1998) present a detailed derivation of the formula for the *PP* reflection coefficient at a weak contrast interface separating two weakly anisotropic media. Here only a short review of basic steps and formulae is made.

We consider a model consisting of two homogeneous weakly anisotropic halfspaces separated by an interface with the unit normal v_i pointing into the halfspace, in which an incident wave propagates. We call it the halfspace 1 and denote its density and the density-normalized elastic parameters $\rho^{(1)}$ and $a^{(1)}_{ijkl}$. The same parameters in the halfspace 2 are denoted $\rho^{(2)}$ and $a^{(2)}_{ijkl}$. The incident and generated waves satisfy the boundary conditions at the interface: continuity of the displacement and the traction vectors. As a consequence of the boundary conditions, we get important relations for the slowness vectors $p_i^{(N)}$ and $p_i^{(0)}$ of the generated and the incident waves and for the reflection/transmission coefficients $U^{(N)}$. The superscripts $N = 1, 2$ and 3 correspond to reflected $S1$, $S2$ and P wave, the superscripts $N = 4$, 5 and 6 correspond to transmitted $S1$, $S2$ and P wave. The relation for the slowness vectors has the form

$$p_i^{(N)} = b_i + \xi^{(N)} v_i = p_i^{(0)} - (p_k^{(0)} v_k) v_i + \xi^{(N)} v_i, \tag{1}$$

where the quantity $\xi^{(N)}$ can be determined from the polynomial equation of the sixth order

$$\det[a_{ijkl}(b_j + \xi v_j)(b_l + \xi v_l) - \delta_{ik}] = 0. \tag{2}$$

The R/T coefficients $U^{(N)}$ are determined by solving the system of six algebraic equations

$$\begin{aligned} U^{(1)} g_i^{(1)} + U^{(2)} g_i^{(2)} + U^{(3)} g_i^{(3)} - U^{(4)} g_i^{(4)} - U^{(5)} g_i^{(5)} - U^{(6)} g_i^{(6)} &= -g_i^{(0)}, \\ U^{(1)} X_i^{(1)} + U^{(2)} X_i^{(2)} + U^{(3)} X_i^{(3)} - U^{(4)} X_i^{(4)} - U^{(5)} X_i^{(5)} - U^{(6)} X_i^{(6)} &= -X_i^{(0)}, \end{aligned} \tag{3}$$

where

$$\begin{aligned} X_i^{(N)} &= \rho^{(1)} a^{(1)}_{ijkl} v_j g_k^{(N)} p_l^{(N)}, \quad N = 0, 1, 2, 3, \\ X_i^{(N)} &= \rho^{(2)} a^{(2)}_{ijkl} v_j g_k^{(N)} p_l^{(N)}, \quad N = 4, 5, 6. \end{aligned} \tag{4}$$

The vectors $X_i^{(N)}$ are the amplitude-normalized traction vectors. Equation (3) can be rewritten into the matrix form

$$C_{\alpha\beta} U_\beta = B_\alpha, \tag{5}$$

where $C_{\alpha\beta}$ is the displacement-stress matrix of the R/T waves, U_β is the vector of the R/T coefficients and B_α is the amplitude-normalized displacement-stress vector of the incident wave.

In each halfspace, the density-normalized elastic parameters and the density are considered in the form

$$a_{ijkl}^{(I)} = a_{ijkl}^{(I)0} + \delta a_{ijkl}^{(I)}, \quad \rho^{(I)} = \rho^{(I)0} + \delta\rho^{(I)}, \quad I = 1, 2. \tag{6}$$

The symbols $a_{ijkl}^{(I)0}$ and $\rho^{(I)0}$ denote the elastic parameters and the density of the background isotropic media in both halfspaces. The quantities $\delta a_{ijkl}^{(I)}$ and $\delta\rho^{(I)}$ represent small deviations from the isotropic backgrounds.

We now linearize Eq. (5) with respect to the deviations of elastic parameters $a_{ijkl}^{(I)}$ and the density $\rho^{(I)}$ from the average values $\bar{a}_{ijkl}^{0}$ and $\bar{\rho}^{0}$ of the parameters and the density of the isotropic backgrounds. The average value $\bar{w}$ of parameters $w^{(I)}$ is defined as follows

$$\bar{w} = \tfrac{1}{2}(w^{(1)} + w^{(2)}). \tag{7}$$

In this way, we get

$$C_{\alpha\beta}^{0}\delta U_{\beta} = \delta B_{\alpha} - \delta C_{\alpha\beta} U_{\beta}^{0}. \tag{8}$$

The symbols $C_{\alpha\beta}^{0}$ and U_{β}^{0} denote the matrix $C_{\alpha\beta}$ and the vector U_{β}, specified for a fictitious interface in a continuous isotropic space characterized by the parameters $\bar{a}_{ijkl}^{0}$ and the density $\bar{\rho}^{0}$. For the incident wave with a unit amplitude, the vector δU_{α} contains perturbations of three reflection and three transmission coefficients from their values U_{α}^{0} in the background isotropic medium. The vector δB_{α} and the matrix $\delta C_{\alpha\beta}$ are the perturbations of the corresponding vector and matrix in Eq. (5). The linearized reflection/transmission coefficients can be sought in the form $U_{\alpha}^{0} + \delta U_{\alpha}$, where δU_{α} is given as

$$\delta U_{\alpha} = (\mathbf{C^0})_{\alpha\beta}^{-1}(\delta B_{\beta} - \delta C_{\beta\gamma} U_{\gamma}^{0}). \tag{9}$$

The basic step in making Eq. (9) useful is the inversion of the matrix $C_{\beta\gamma}^{0}$. Vavryčuk and Pšenčík (1997) found the inverted matrix in the form

$$(\mathbf{C^0})_{\alpha\beta}^{-1} = \begin{pmatrix}
-\dfrac{\bar{\beta}^2 Y p_1^0 \cos\Psi}{Z_S} & \dfrac{\sin\Psi}{2} & -p_1^0\bar{\beta}\cos\Psi & \dfrac{\cos\Psi}{2\bar{\beta}\bar{\rho}^0} & -\dfrac{p_1^0\bar{\beta}\sin\Psi}{Z_S} & \dfrac{\bar{\beta}^2(p_1^0)^2\cos\Psi}{Z_S} \\[2ex]
\dfrac{\bar{\beta}^2 Y p_1^0 \sin\Psi}{Z_S} & \dfrac{\cos\Psi}{2} & p_1^0\bar{\beta}\sin\Psi & -\dfrac{\sin\Psi}{2\bar{\rho}^0\bar{\beta}} & -\dfrac{\bar{\beta}p_1^0\cos\Psi}{Z_S} & -\dfrac{\bar{\beta}^2(p_1^0)^2\sin\Psi}{Z_S} \\[2ex]
\dfrac{\bar{\beta}^2 p_1^0}{\bar{\alpha}} & 0 & -\dfrac{p_1^0\bar{\beta}^2 Y}{Z_P} & -\dfrac{\bar{\beta}^2(p_1^0)^2}{Z_P} & 0 & \dfrac{1}{2\bar{\rho}^0\bar{\alpha}} \\[2ex]
-\dfrac{\bar{\beta}^2 Y p_1^0 \cos\Phi}{Z_S} & -\dfrac{\sin\Phi}{2} & p_1^0\bar{\beta}\cos\Phi & -\dfrac{\cos\Phi}{2\bar{\beta}\rho^0} & -\dfrac{p_1^0\bar{\beta}\sin\Phi}{Z_S} & \dfrac{\bar{\beta}^2(p_1^0)^2\cos\Phi}{Z_S} \\[2ex]
\dfrac{\bar{\beta}^2 Y p_1^0 \sin\Phi}{Z_S} & -\dfrac{\cos\Phi}{2} & -p_1^0\bar{\beta}\sin\Phi & \dfrac{\sin\Phi}{2\bar{\rho}^0\bar{\beta}} & -\dfrac{\bar{\beta}p_1^0\cos\Phi}{Z_S} & -\dfrac{\bar{\beta}^2(p_1^0)^2\sin\Phi}{Z_S} \\[2ex]
-\dfrac{\bar{\beta}^2 p_1^0}{\bar{\alpha}} & 0 & -\dfrac{p_1^0\bar{\beta}^2 Y}{Z_P} & -\dfrac{\bar{\beta}^2(p_1^0)^2}{Z_P} & 0 & -\dfrac{1}{2\bar{\rho}^0\bar{\alpha}}
\end{pmatrix}, \tag{10}$$

where

$$Y = \bar{\rho}^0(1 - 2\bar{\beta}^2(p_1^0)^2), \quad Z_P = 2\bar{\alpha}\bar{\rho}^0\bar{\beta}^2 p_1^0 p_3^{0P}, \quad Z_S = 2\bar{\rho}^0\bar{\beta}^3 p_1^0 p_3^{0S}. \tag{11}$$

Here p_1^0, p_3^{0P} and p_3^{0S} denote the x and z components of the slowness vector of the P and S waves in the background isotropic medium. The angles Ψ and Φ are angles of rotation of the polarization vectors of reflected (Ψ) and transmitted (Φ) S waves in the planes perpendicular to their rays. The vectors must be rotated in order to guarantee a small perturbation from the isotropic to the weakly anisotropic medium. The determination of the angles Ψ and Φ, of course, complicates the procedure of the determination of the linearized R/T coefficients. VAVRYČUK and PŠENČÍK (1998) show that this is not the case for the *PP* reflection and transmission coefficients. In the following, we concentrate on these two coefficients. Derivation of the formulae for the converted waves is left for a next study.

3. PP Wave Displacement Coefficients of Reflection and Transmission

The elastic parameters and the density of the isotropic backgrounds in both halfspaces can be chosen arbitrarily but they should not deviate much from the elastic parameters and the density of the weakly anisotropic media in the half-spaces. We choose the elastic parameters in such a way that our results are simply reducible to the results of previous authors. Specifically, we choose the P- and S-wave velocities $\alpha^{(I)}$ and $\beta^{(I)}$ as follows

$$(\alpha^{(I)})^2 = A_{33}^{(I)}, \quad (\beta^{(I)})^2 = A_{55}^{(I)}. \tag{12}$$

In addition to $\alpha^{(I)}$, $\beta^{(I)}$ and $\rho^{(I)}$, we introduce the P-wave impedance $Z^{(I)}$ and the shear modulus $G^{(I)}$,

$$Z^{(I)} = \rho^{(I)}\alpha^{(I)}, \quad G^{(I)} = \rho^{(I)}(\beta^{(I)})^2. \tag{13}$$

The contrast of a parameter w across the interface is denoted by Δw and it is defined as follows

$$\Delta w = w^{(2)} - w^{(1)}. \tag{14}$$

As VAVRYČUK and PŠENČÍK (1998), we specify the R/T coefficients by the direction of the phase normal n_i of the incident plane wave, specifically by the angles of the incidence θ and the azimuth φ

$$n_i \equiv (\cos\varphi \sin\theta, \sin\varphi \sin\theta, \cos\theta)^T. \tag{15}$$

The approximate formula for the *PP* reflection coefficient $R_{PP}(\varphi, \theta)$ at an interface separating two weakly but arbitrarily anisotropic media can be obtained from Eq. (9) and has the form

$$R_{PP}(\varphi,\theta) = R_{PP}^{iso}(\theta) + \frac{1}{2}\left[\Delta\delta_x \cos^2\varphi + \left(\Delta\delta_y - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma\right)\sin^2\varphi \right.$$
$$\left. + 2\left(\Delta\chi_z - 4\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\left(\frac{A_{45}}{A_{55}}\right)\right)\cos\varphi\sin\varphi\right]\sin^2\theta$$
$$+ \tfrac{1}{2}[\Delta\varepsilon_x \cos^4\varphi + \Delta\varepsilon_y \sin^4\varphi + \Delta\delta_z \cos^2\varphi\sin^2\varphi$$
$$+ 2(\Delta\varepsilon_{16}\cos^2\varphi + \Delta\varepsilon_{26}\sin^2\varphi)\sin\varphi\cos\varphi]\sin^2\theta\tan^2\theta. \qquad (16)$$

The symbol $R_{PP}^{iso}(\theta)$ in Eq. (16) denotes the weak contrast reflection coefficient at an interface separating two slightly different isotropic media, see e.g. AKI and RICHARDS (1980):

$$R_{PP}^{iso}(\theta) = \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}\left[\frac{\Delta\alpha}{\bar{\alpha}} - 4\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2\frac{\Delta G}{\bar{G}}\right]\sin^2\theta + \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}\sin^2\theta\tan^2\theta. \qquad (17)$$

In addition to averages and differences of the parameters $\rho^{(I)}$, $\alpha^{(I)}$ and $\beta^{(I)}$ of the background media and the angles θ and φ, the reflection coefficient depends on contrasts of eight *weak anisotropy* (WA) parameters (see PŠENČÍK and GAJEWSKI, 1996) describing the P-wave phase velocity and polarization in weakly anisotropic media. The WA parameters characterize deviations of properties of the studied medium from the isotropic background. In isotropic media, the WA parameters become zero. The total number of the WA parameters is 15. For the P-wave velocity α specified by Eq. (12), their number reduces to 14 and they are, see PŠENČÍK and GAJEWSKI (1998):

$$\delta_x = \frac{A_{13}+2A_{55}-A_{33}}{A_{33}}, \quad \delta_y = \frac{A_{23}+2A_{44}-A_{33}}{A_{33}}, \quad \delta_z = \frac{A_{12}+2A_{66}-A_{33}}{A_{33}},$$
$$\chi_x = \frac{A_{14}+2A_{56}}{A_{33}}, \quad \chi_y = \frac{A_{25}+2A_{46}}{A_{33}}, \quad \chi_z = \frac{A_{36}+2A_{45}}{A_{33}},$$
$$\varepsilon_{15} = \frac{A_{15}}{A_{33}}, \quad \varepsilon_{16} = \frac{A_{16}}{A_{33}}, \quad \varepsilon_{24} = \frac{A_{24}}{A_{33}}, \quad \varepsilon_{26} = \frac{A_{26}}{A_{33}}, \quad \varepsilon_{34} = \frac{A_{34}}{A_{33}}, \quad \varepsilon_{35} = \frac{A_{35}}{A_{33}},$$
$$\varepsilon_x = \frac{A_{11}-A_{33}}{2A_{33}}, \quad \varepsilon_y = \frac{A_{22}-A_{33}}{2A_{33}}, \quad \gamma = \frac{A_{44}-A_{55}}{2A_{55}}, \qquad (18)$$

Note that in addition to the P-wave WA parameters, the reflection coefficient also depends on the parameters A_{45}/A_{55} and γ. The symbol γ denotes the "shear-wave splitting parameter" introducted by THOMSEN (1986), see also RUEGER (1996). It is also of interest to note that the above reflection coefficient is reciprocal, i.e. it yields the same value for angles θ, φ and θ, $\varphi+\pi$. This indicates that the PP displacement coefficient of reflection is proportional to the PP coefficient of reflection of the square root of the vertical energy flux, see AKI and RICHARDS (1980) and CHAPMAN (1994).

Using the same approach as for the derivation of the reflection coefficient, we obtain the formula for the *PP* displacement coefficient of transmission. It reads

$$\begin{aligned} T_{PP}(\varphi,\theta) = {} & T^{iso}_{PP}(\theta) + \tfrac{1}{2}[\Delta\delta_x \cos^2\varphi + \Delta\delta_y \sin^2\varphi + 2\Delta\chi_z \cos\varphi \sin\varphi] \sin^2\theta \\ & + 2[\Delta\varepsilon_{35}\cos\varphi + \Delta\varepsilon_{34}\sin\varphi - \Delta\chi_x \cos^2\varphi \sin\varphi - \Delta\chi_y \cos\varphi \sin^2\varphi \\ & - \Delta\varepsilon_{15}\cos^3\varphi - \Delta\varepsilon_{24}\sin^3\varphi] \sin^3\theta \cos\theta \\ & + \tfrac{1}{2}[\Delta\varepsilon_x \cos^4\varphi + \Delta\varepsilon_y \sin^4\varphi + \Delta\delta_z \cos^2\varphi \sin^2\varphi \\ & + 2(\Delta\varepsilon_{16}\cos^2\varphi + \Delta\varepsilon_{26}\sin^2\varphi)\sin\varphi\cos\varphi] \sin^2\theta \tan^2\theta \\ & + [\Delta\varepsilon_x \cos^4\varphi + \Delta\varepsilon_y \sin^4\varphi + \Delta\delta_z \cos^2\varphi \sin^2\varphi \\ & + 2(\Delta\varepsilon_{16}\cos^2\varphi + \Delta\varepsilon_{26}\sin^2\varphi)\sin\varphi\cos\varphi \\ & - \Delta\delta_x \cos^2\varphi - \Delta\delta_y \sin^2\varphi - 2\Delta\chi_z \cos\varphi \sin\varphi] \sin^4\theta. \end{aligned} \tag{19}$$

The symbol $T^{iso}_{PP}(\theta)$ denotes the weak contrast transmission coefficient at an interface separating two isotropic media, see again AKI and RICHARDS (1980):

$$T^{iso}_{PP}(\theta) = 1 - \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}} \tan^2\theta. \tag{20}$$

The meaning of the other symbols is the same as in (12)–(14) and (18).

By comparing formulae (16) and (19), we can see that the *PP* reflection coefficient depends on contrast of only 8 *P*-wave WA parameters while the *PP* transmission coefficient depends on contrast of the complete set of the WA parameters. The eight WA parameters are the coefficients of the azimuthally symmetric terms of the expressions for the *P*-wave phase velocity and polarization in weakly anisotropic media, see PŠENČÍK and GAJEWSKI (1998). The remaining WA parameters, which are the coefficients of the azimuthally antisymmetric terms, disappear in (16) due to the symmetry of the unconverted *PP* reflection. We can also see that the reflection coefficient contains some information on the vertical shear-wave propagation, see the "shear-wave splitting parameter" γ in (16), while no such information appears in the formula for the transmission coefficient. In contrast to the reflection coefficient, the transmission coefficient is not reciprocal. This indicates that the relation of the displacement coefficient of transmission to the coefficient related to vertical energy flux is more complicated than in the case of reflection. It confirms the well-known fact that the displacement coefficients are generally not reciprocal.

4. PP Wave R/T Coefficients for Transversely Isotropic Media with a Horizontal Axis of Symmetry along x Axis

The R/T coefficients (16) and (19) can be applied to any type of anisotropic media in both halfspaces surrounding the interface. They simplify considerably if higher symmetry anisotropy is considered. Let us, for example, consider that both halfspaces are transversely isotropic with horizontal axes of symmetry along the x axis. This kind of anisotropy is very important since it describes effects of a system of parallel vertical cracks. For simplicity we consider that the axes of symmetry in both media are parallel. For such a case, the non-zero density-normalized elastic parameters satisfy the following relations on both sides of the interface

$$A_{33} = A_{22}, \quad A_{66} = A_{55}, \quad A_{13} = A_{12}, \quad A_{23} = A_{33} - 2A_{44}. \tag{21}$$

For such a specification, the formulae (16) and (19) reduce to

$$R_{PP}(\varphi, \theta) = R_{PP}^{iso}(\theta) + \frac{1}{2}\left[\Delta\delta_x \cos^2\varphi - 8\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2 \Delta\gamma \sin^2\varphi\right]\sin^2\theta$$
$$+ \tfrac{1}{2}(\Delta\varepsilon_x \cos^2\varphi + \Delta\delta_x \sin^2\varphi)\cos^2\varphi \sin^2\theta \tan^2\theta \tag{22}$$

and

$$T_{PP}(\varphi, \theta) = T_{PP}^{iso}(\theta) + \tfrac{1}{2}\Delta\delta_x \cos^2\varphi \sin^2\theta$$
$$+ \tfrac{1}{2}(\Delta\varepsilon_x \cos^2\varphi + \Delta\delta_x \sin^2\varphi)\cos^2\varphi \sin^2\theta \tan^2\theta$$
$$+ (\Delta\varepsilon_x - \Delta\delta_x)\cos^4\varphi \sin^4\theta. \tag{23}$$

The symbols δ_x, ε_x and γ are given in Eqs. (18).

5. Test Example

To test the accuracy of the approximate formulae for the R/T coefficients, we first use the same models as in VAVRYČUK and PŠENČÍK (1998). The halfspace, in which the incident wave propagates, is isotropic, the other halfspace is HTI with the axis of symmetry along the x axis. For these models we calculate values of the R/T coefficients $R_{PP}(\varphi, \theta)$ and $T_{PP}(\varphi, \theta)$ using numerical solution of boundary conditions and compare them with values calculated using the approximate formulae (22) and (23). For the isotropic overburden, the formulae (22) and (23) reduce to

$$R_{PP}(\varphi, \theta) = R_{PP}^{iso}(\theta) + \frac{1}{2}\left[\delta_x^{(2)} \cos^2\varphi - 8\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2 \gamma^{(2)} \sin^2\varphi\right]\sin^2\theta$$
$$+ \tfrac{1}{2}(\varepsilon_x^{(2)} \cos^4\varphi + \delta_x^{(2)} \cos^2\varphi \sin^2\varphi)\sin^2\theta \tan^2\theta \tag{24}$$

and

$$T_{PP}(\varphi, \theta) = T_{PP}^{iso}(\theta) + \tfrac{1}{2}\delta_x^{(2)} \cos^2 \varphi \sin^2 \theta$$
$$+ \tfrac{1}{2}(\varepsilon_x^{(2)} \cos^2 \varphi + \delta_x^{(2)} \sin^2 \varphi) \cos^2 \varphi \sin^2 \theta \tan^2 \theta$$
$$+ (\varepsilon_x^{(2)} - \delta_x^{(2)}) \cos^4 \varphi \sin^4 \theta. \tag{25}$$

Note that for the models with isotropic overburden, the R/T coefficients depend directly on WA parameters of the anisotropic halfspace.

Two isotropic and two TI halfspaces are considered. The P- and S-wave velocities in the isotropic halfspaces are (A): $\alpha = 4.0$ km/sec, $\beta = 2.31$ km/sec and $\rho = 2.65$ g/cm^3; (B): $\alpha = 3.0$ km/sec, $\beta = 1.73$ km/sec and $\rho = 2.2$ g/cm^3. Anisotropy of the anisotropic halfspaces is assumed to be caused by a system of vertical parallel dry cracks, see HUDSON (1981). The P- and S-wave velocities of the host rock are 4.0 km/sec and 2.31 km/sec and the density is 2.6 g/cm^3. The aspect ratio is 10^{-4} and the crack densities are (C): 0.05 and (D): 0.1. The corresponding matrices of the density-normalized elastic parameters (in GPa) with the axis of symmetry along the x axis have the form

$$\begin{pmatrix} 11.96 & 3.99 & 3.99 & 0.00 & 0.00 & 0.00 \\ & 15.55 & 4.88 & 0.00 & 0.00 & 0.00 \\ & & 15.55 & 0.00 & 0.00 & 0.00 \\ & & & 5.33 & 0.00 & 0.00 \\ & & & & 4.76 & 0.00 \\ & & & & & 4.76 \end{pmatrix}$$

in the case C and

$$\begin{pmatrix} 9.43 & 3.14 & 3.14 & 0.00 & 0.00 & 0.00 \\ & 15.27 & 4.60 & 0.00 & 0.00 & 0.00 \\ & & 15.27 & 0.00 & 0.00 & 0.00 \\ & & & 5.33 & 0.00 & 0.00 \\ & & & & 4.25 & 0.00 \\ & & & & & 4.25 \end{pmatrix}$$

in the case D. Sections of the phase velocity surfaces with the vertical plane containing axes of symmetry for the cases C ($e = 0.05$) and D ($e = 0.1$) are shown in Figure 1.

We consider three models a, b and c, see Table 1. In the models a and b, the phase velocity of the halfspace 1 is for all azimuths higher than the phase velocity in the halfspace 2. In the model c, the relation is opposite. In all cases, the values of reflection coefficients start to rise considerably for higher angles of incidence and the approximate formulae of this paper become inapplicable. From this reason, we consider the angles of incidence only in the interval (0°, 42°). The values of the velocities and the density of the background isotropic medium were determined from formulae (7) and (12). For the used values, see the figure captions.

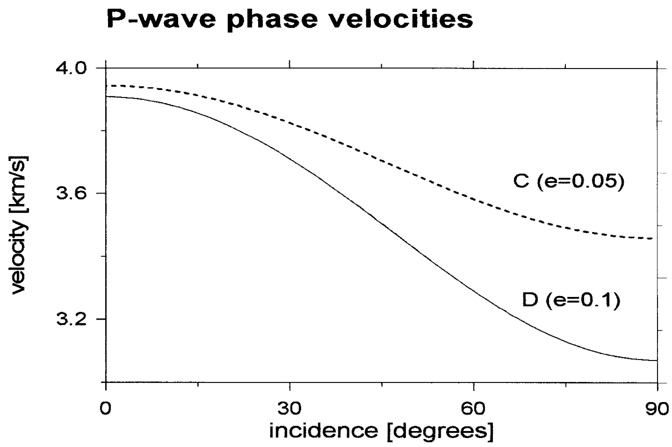

Figure 1
Phase velocity sections with the vertical plane containing the symmetry axes of two "dry crack" models with crack densities $e = 0.05$ (C) and $e = 0.1$ (D).

The test results are displayed in the form of four plots in Figures 2–7. In all the plots the horizontal axis corresponds to the angle of incidence θ, measured in degrees. The vertical axis corresponds to the azimuth φ, also in degrees. Azimuth $\varphi = 0°$ corresponds to the profile along the axis of symmetry, azimuth $\varphi = 90°$ corresponds to the profile in the plane perpendicular to the axis of symmetry, i.e., in the isotropy plane.

The tests of accuracy of the reflection coefficient have been discussed in VAVRYČUK and PŠENČÍK (1998). For completeness, we show in Figures 2–4 only the plots of these results in a different display. Note that in contrast to results discussed in the above-mentioned paper, no shift of values of approximate coefficients is made here. Figures 5–7 contain similar plots as Figures 2–4 but for the transmission coefficients. They are self-explanatory.

A general feature of the presented numerical examples is a higher relative accuracy of the *PP* transmission coefficients compared to the reflection coefficients.

Table 1

Models used in test examples. I—isotropic, HTI—transversely isotropic (TI) with the horizontal axis of symmetry, VTI—TI with the vertical axis of symmetry, ITI—TI with the inclined axis of symmetry (30° from the horizontal in the (x, z) plane). For the description of parameters of models see the text

	a	b	c	d	e
Halfspace 1	A (I)	A (I)	B (I)	E (VTI)	E (VTI)
Halfspace 2	C (HTI)	D (HTI)	D (HTI)	F (HTI)	F (ITI)

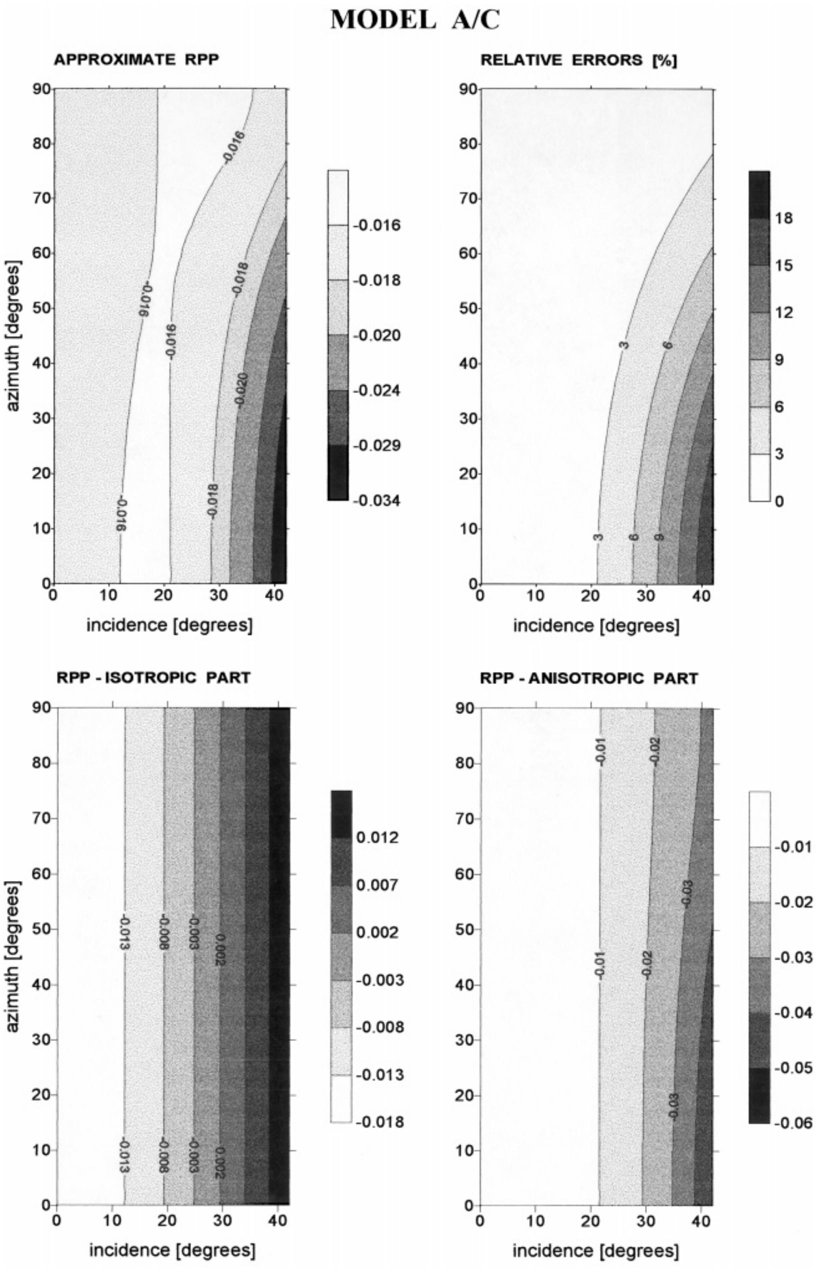

Figure 2

The maps of the approximate R_{PP} and R_{PP}^{iso} reflection coefficients (upper and bottom left), the map of relative errors of the approximate coefficient (upper right) and the map of the difference $R_{PP} - R_{PP}^{iso}$ for the model *a* of the Table 1. The isotropic halfspace (A): $\alpha = 4.0$ km/sec, $\beta = 2.31$ km/sec, $\rho = 2.65$ g/cm^3. The HTI halfspace (C): axis of symmetry along x axis, host rock: $\alpha = 4.0$ km/sec, $\beta = 2.31$ km/sec, $\rho = 2.60$ g/cm^3; dry cracks: aspect ratio $a = 0.0001$, crack density $e = 0.05$. Isotropic background: $\bar{\alpha} = 3.97$ km/sec, $\bar{\beta} = 2.25$ km/sec, $\bar{\rho} = 2.63$ g/cm^3.

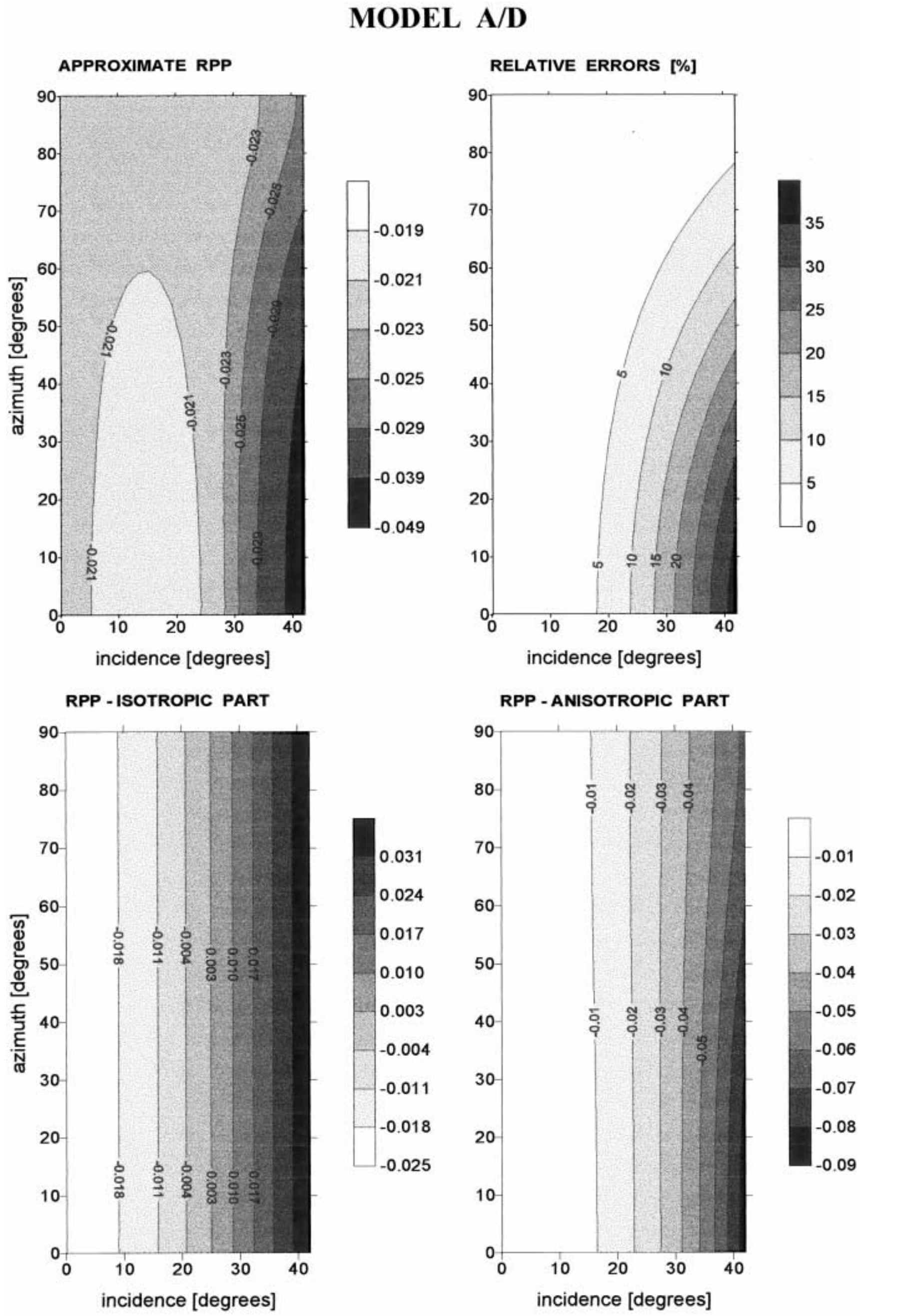

Figure 3

The same as in Figure 2 but for the model b of the Table 1. The isotropic halfspace (A): $\alpha = 4.0$ km/sec, $\beta = 2.31$ km/sec, $\rho = 2.65$ g/cm^3. The HTI halfspace (D): axis of symmetry along x axis, host rock: $\alpha = 4.0$ km/sec, $\beta = 2.31$ km/sec, $\rho = 2.60$ g/cm^3; dry cracks: aspect ratio $a = 0.0001$, crack density $e = 0.01$. Isotropic background: $\bar{\alpha} = 3.95$ km/sec, $\bar{\beta} = 2.19$ km/sec, $\bar{\rho} = 2.63$ g/cm^3.

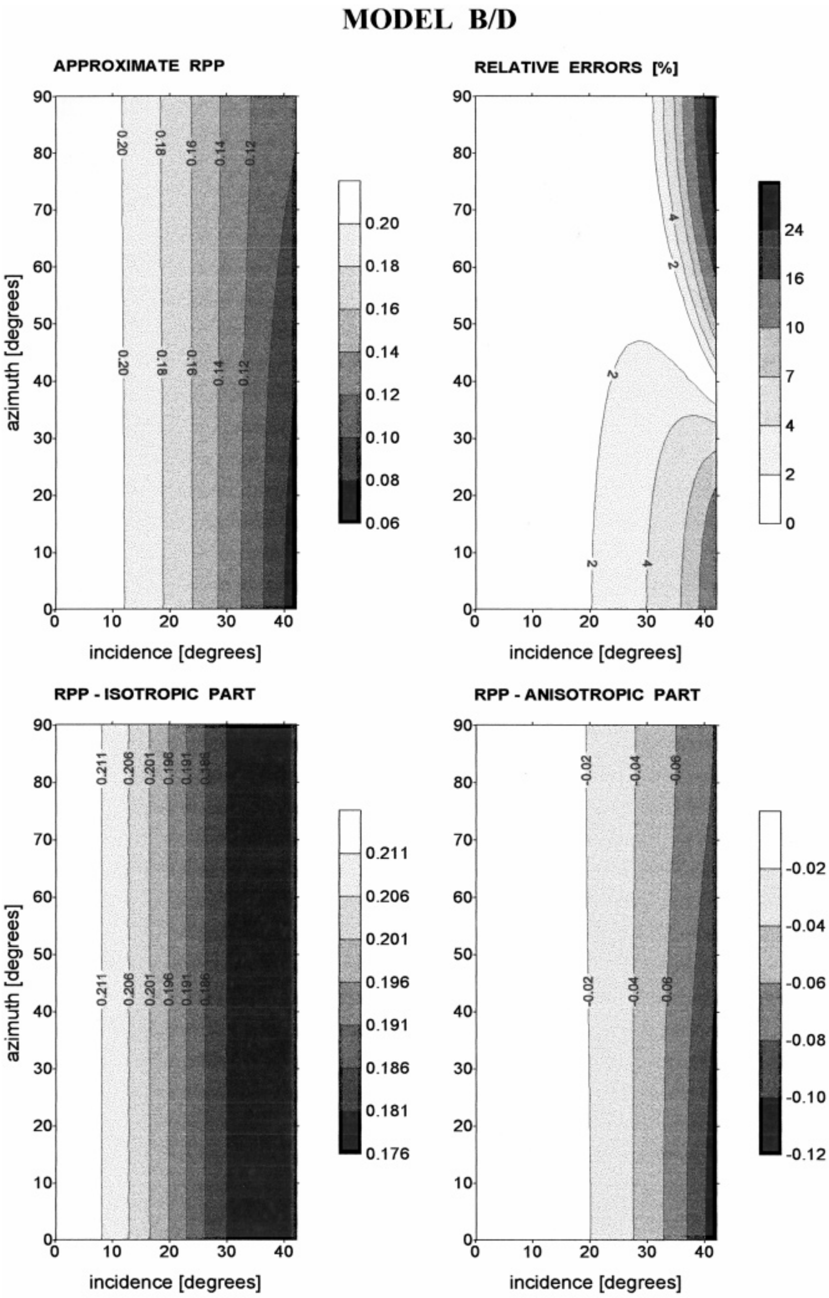

Figure 4

The same as in Figure 2 but for the model *c* of the Table 1. The isotropic halfspace (B): $\alpha = 3.0$ km/sec, $\beta = 1.73$ km/sec, $\rho = 2.2$ g/cm^3. The HTI halfspace (D): axis of symmetry along x axis, host rock: $\alpha = 4.0$ km/sec, $\beta = 2.31$ km/sec, $\rho = 2.60$ g/cm^3; dry cracks: aspect ratio $a = 0.0001$, crack density $e = 0.1$. Isotropic background: $\bar{\alpha} = 3.45$ km/sec, $\bar{\beta} = 1.90$ km/sec, $\bar{\rho} = 2.4$ g/cm^3.

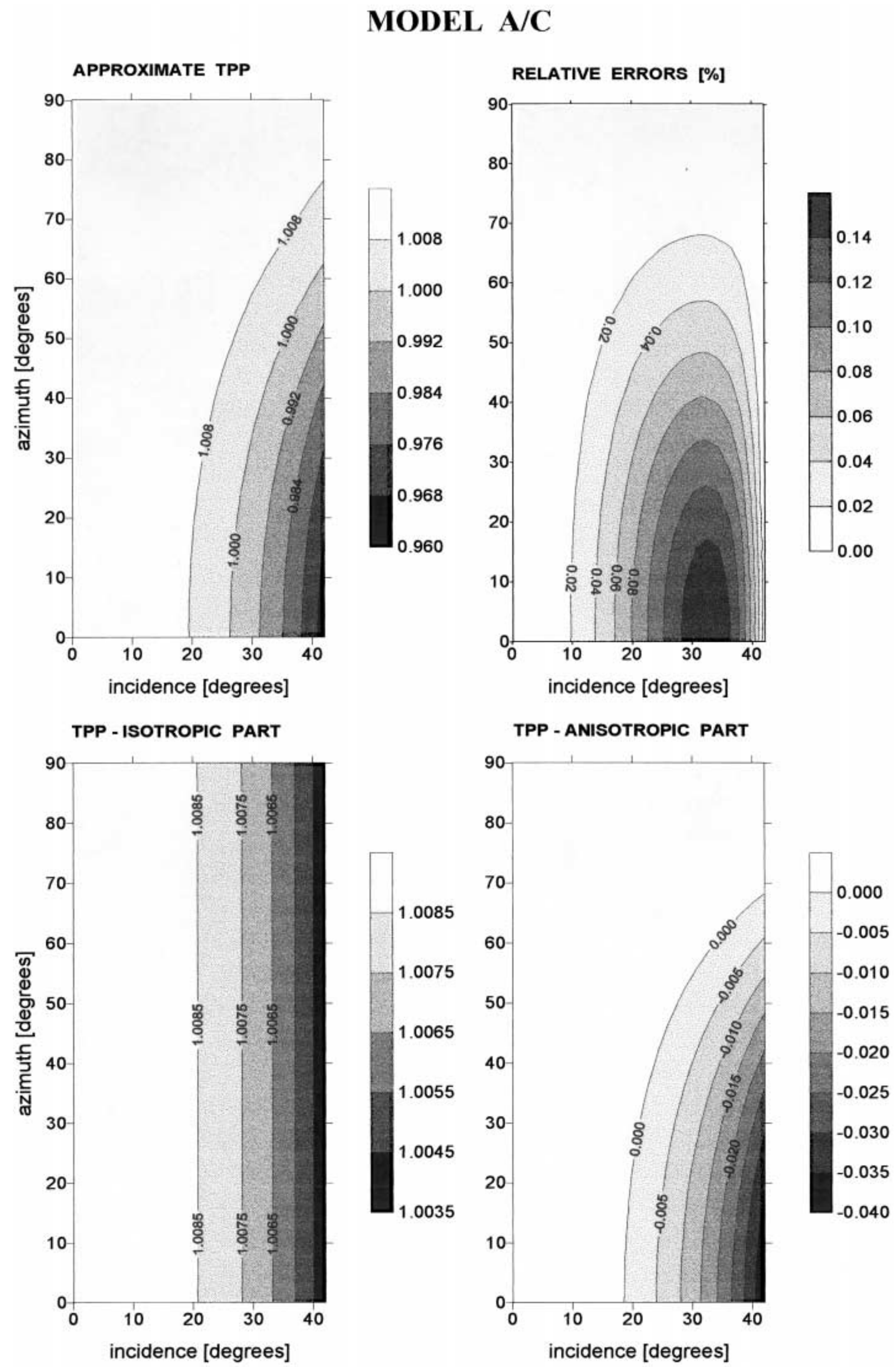

Figure 5
The same as in Figure 2 but for the transmission coefficient T_{PP}.

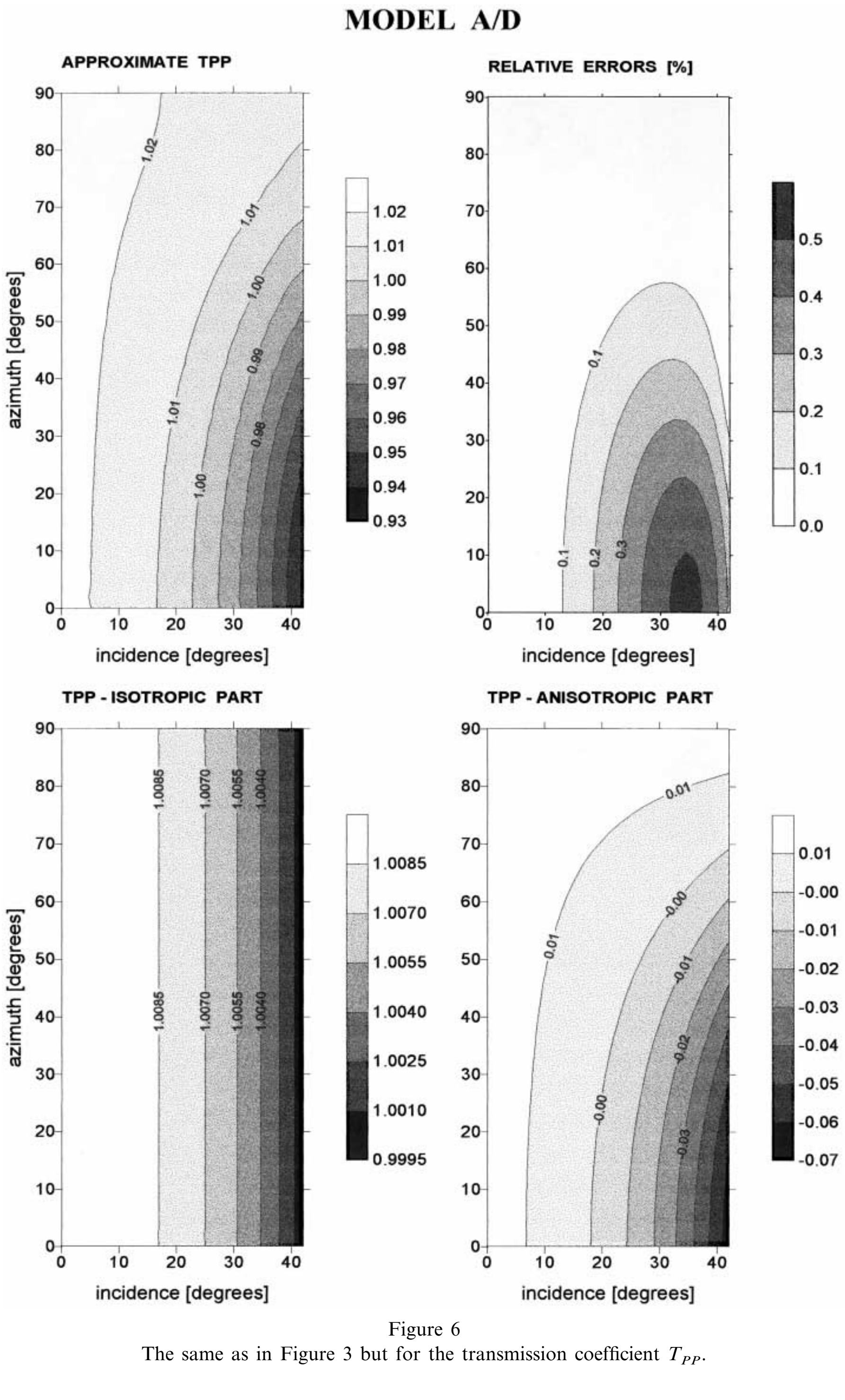

Figure 6
The same as in Figure 3 but for the transmission coefficient T_{PP}.

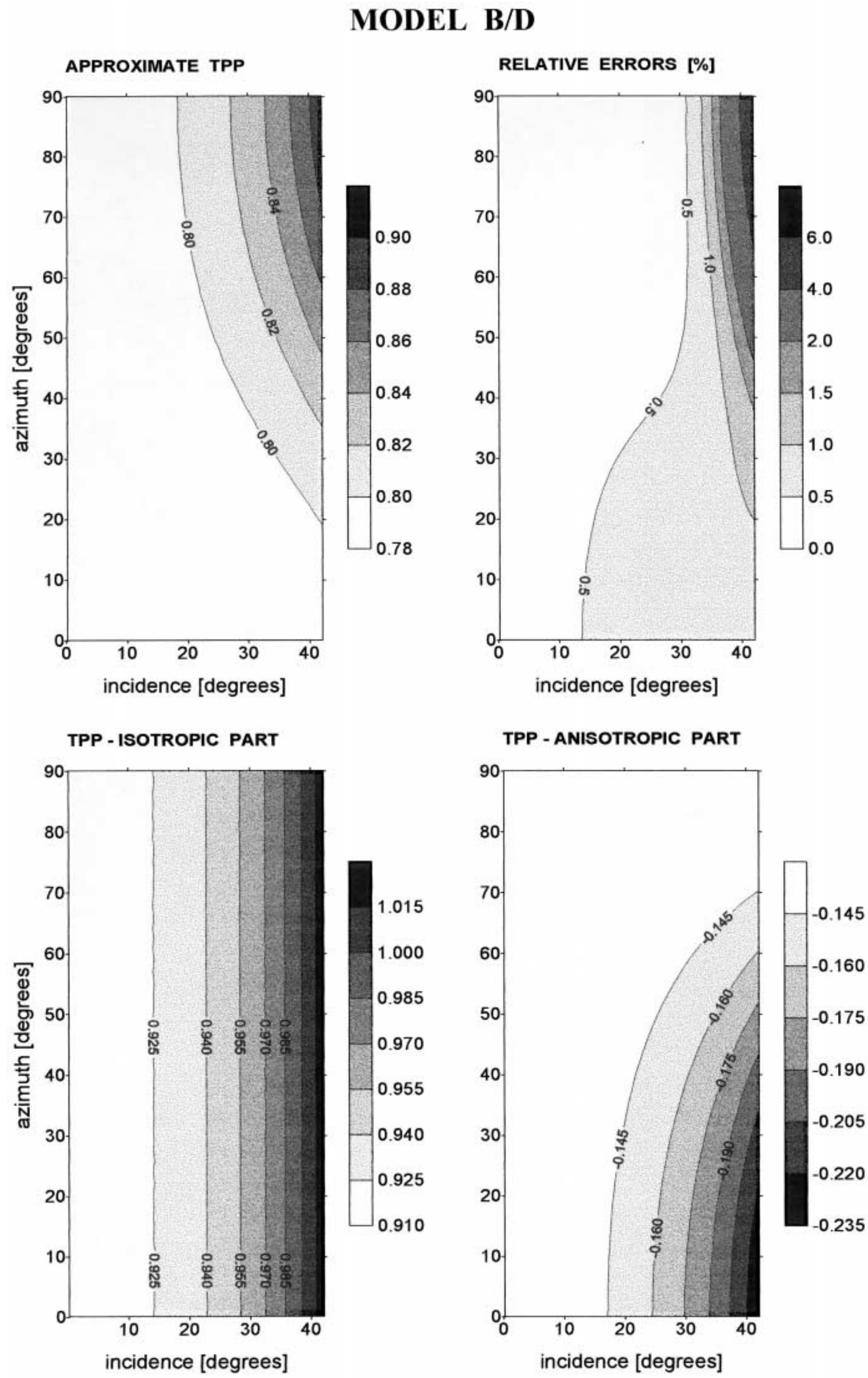

Figure 7
The same as in Figure 4 but for the transmission coefficient T_{PP}.

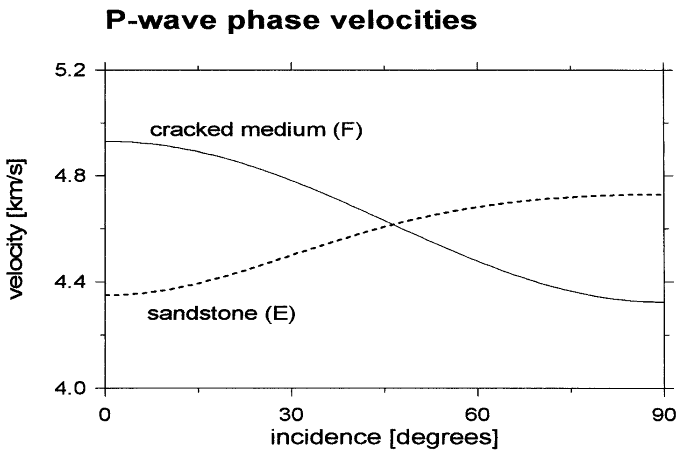

Figure 8
Phase velocity sections of the Mesaverde sandstone ((E)—dashed line) and the "dry crack" model ((F)—full line) with the vertical plane containing the symmetry axes.

It is due to the fact that the reflection coefficients are quite small while the transmission coefficients are close to unity and only vary slightly. We can see that the accuracy of coefficients remains high even in Figures 4 and 7, which correspond to a rather strong contrast (up to 25%) and strong anisotropy (nearly 20%).

Finally, we present the approximate R_{PP} and T_{PP} coefficients for more complicated models than considered before. Figure 8 shows phase velocity sections of two TI materials, E and F, filling the halfspaces in these models. The halfspace 1, in which incident wave propagates, is filled by Mesaverde immature sandstone (E), see THOMSEN (1986), which is VTI. Its matrix of density-normalized elastic parameters (in GPa) has the form

$$\begin{pmatrix} 22.36 & 6.36 & 8.49 & 0.00 & 0.00 & 0.00 \\ & 22.36 & 8.49 & 0.00 & 0.00 & 0.00 \\ & & 18.91 & 0.00 & 0.00 & 0.00 \\ & & & 6.61 & 0.00 & 0.00 \\ & & & & 6.61 & 0.00 \\ & & & & & 8.00 \end{pmatrix}$$

and the density is $\rho = 2.46$ g/cm^3.

The halfspace 2 is filled by a system of parallel dry cracks (F). The cracks are vertical so that the halfspace 2 is HTI with the axis of symmetry parallel to the x axis. The matrix of density-normalized elastic parameters (in GPa) has the form

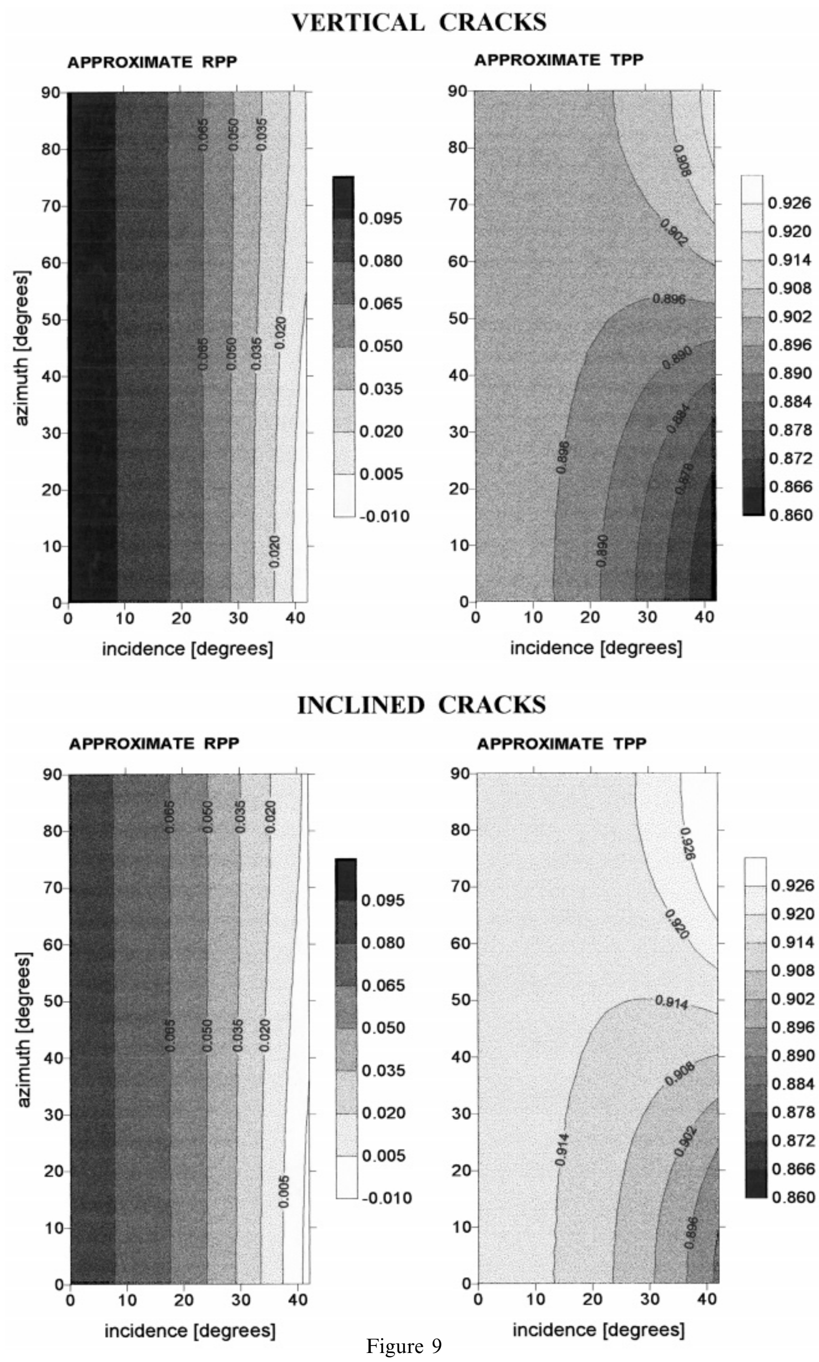

Figure 9

The maps of the approximate R_{PP} and T_{PP} coefficients for the models d and e of Table 1. The upper halfspace (E): the VTI Mesaverde sandstone. The lower halfspace: the dry vertical cracks (model d: upper pictures) exhibiting HTI with the axis of symmetry along the x axis and the dry inclined cracks (model e: bottom pictures) exhibiting ITI with the axis of symmetry making the angle of 30° with the x axis within the (x, z) plane. For parameters of both halfspaces, see the text.

$$\begin{pmatrix} 18.69 & 6.20 & 6.20 & 0.00 & 0.00 & 0.00 \\ & 24.31 & 7.60 & 0.00 & 0.00 & 0.00 \\ & & 24.31 & 0.00 & 0.00 & 0.00 \\ & & & 8.35 & 0.00 & 0.00 \\ & & & & 7.45 & 0.00 \\ & & & & & 7.45 \end{pmatrix}$$

and the density is $\rho = 2.65$ g/cm^3. In addition to vertical cracks, cracks inclined by 30° are also considered. This configuration corresponds to the axis of symmetry making the angle of 30° with the x axis in the plane (x, z).

The upper pictures in Figure 9 illustrate the approximate R_{PP} and T_{PP} coefficients in the model *d*, see Table 1, with vertical cracks in the halfspace 2. The bottom pictures in Figure 9 display the same coefficients for the model *e* with the inclined cracks in the halfspace 2. We can see that the inclination of cracks has greater effects on the transmission coefficient. The transmission coefficient also behaves in both cases more "anisotropically" than the reflection coefficient.

6. Conclusions

Presented approximate plane wave *PP* displacement coefficients of reflection and transmission at a weak contrast interface separating two weakly but generally anisotropic media, give a clear insight into the dependence of these coefficients on parameters of media surrounding the interface. The coefficients consist of two parts. The first part is the coefficient for a weak contrast interface separating two slightly different isotropic media. The second part is due to a perturbation of the isotropic background. In addition to the angles θ and φ, the perturbation depends linearly on the contrasts of WA parameters but not on the parameters themselves. The *PP* reflection coefficient depends on 8; the *PP* transmission coefficient on 14 P-wave WA parameters (for our choice of the background isotropic medium, 14 WA parameters represent a complete set of the parameters describing the P-wave phase velocity in a weakly anisotropic medium, see PŠENČÍK and GAJEWSKI, 1998). We can conclude that similarly as for the phase velocity and polarization vectors of a P wave propagating in a weakly anisotropic medium, the study of R/T coefficients can yield only limited information on the elastic parameters of the halfspaces surrounding the interface. The *PP* reflection coefficient contains information on "shear-wave splitting parameter" γ. The reflection coefficient is reciprocal while the transmission coefficient is not.

Presented tests show very good performance of the approximate formulae in the selected region of angles of incidence (0°, 42°) even in cases of rather strong anisotropy and contrast across the interface. Slightly higher relative errors of reflection coefficients are caused by the fact that the coefficients in the studied region are rather small.

Both presented formulae of reflection and transmission coefficients are relatively simple if we take into account that they describe the case of R/T between two generally anisotropic media. As their simplified forms derived for higher symmetry anisotropic media, they will surely find applications in both forward and inverse seismic modeling.

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