Elastic near-field wave energy radiated by a spherical cavity

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The energy balance of the elastodynamic field generated by a spherical cavity in a homogeneous, isotropic, and unbounded medium is studied. The near-field waves are found to play an essential role in the energy balance, transforming static energy into wave energy and vice versa, thus displaying a surprising ability to transmit wave energy in the direction opposite to wave propagation. The occurrence of such a negative wave energy clarifies the energy balance of sources, in which the waves are associated with energy dissipation rather than radiation.

CONTENTS

I. INTRODUCTION

This Colloquium was inspired by a study of the energy balance in earthquakes. Earthquakes arise to relieve static strains accumulated in the Earth's crust over a long time, radiating their energy suddenly in the form of seismic waves. An earthquake is initiated by a rupture process on a fault, starting with a small area but rapidly expanding to the entire fault. As a result of rupture, one side begins to move along the other, creating a displacement discontinuity. This shear displacement is generally assumed, but is also often observed directly [a typical value of the displacement for a large earthquake is 5 m (Turcotte and Schubert, 1982)]. The evolution of the displacement discontinuity generates a seismic wave, whose wavelength is smaller than, or at most on the same order as, the fault's length. No larger wavelengths are radiated because a rupture front, controlling seismic wave generation, propagates at a rate comparable with the velocity of seismic waves (a few kilometers per second).

The static strain energy released in an earthquake is not concentrated along a fault but throughout the stressed medium surrounding it. Since the density of the strain energy decreases rapidly with the distance from the fault, most of the strain energy lies in a rather small area with a length scale no larger than 2–3 lengths of the fault. A part of the released strain energy is expended in irreversible deformations during rupturing and in friction between the fault's sides (Freund, 1990; Kostrov and Das, 1989), the other part being radiated away into a distant medium. Both parts of the energy are transported by seismic waves: the first, from a surrounding medium to the fault by short-range waves; the second, from the surrounding medium into a distant medium by long-range waves. The short-range waves are called the near-field waves, whose amplitude decreases with the square of $1/r$ or faster; the long-range waves are called the far-field waves, whose amplitude decreases as $1/r$ (Aki and Richards, 1980).

Separation of wave fields into near- and far-field waves is common for general physical fields, including light, sound, or gravitational waves. Ordinary wave theories, however, study only the energy flow of the far-field waves. This approximation is widely applied in seismology (Červený *et al.*, 1977; Rudnicki and Freund, 1981; Rudnicki, 1983; Madariaga, 1986) because (1) the far-field approximation makes the problem much simpler. (2) The far-field approximation gives reasonable results for a wide range of distances from a source.

The present study develops a new approach to elastic energy flow by taking the near-field waves into account. This approach applies to all distances including the near-source zone (i.e., sufficiently close to the source) where novel features unfamiliar to ordinary wave theory appear. Instead of considering the actual geometry of earthquakes, we shall simplify the problem, adopting as a source model a spherical cavity situated in a perfectly elastic, homogeneous, isotropic, and unbounded medium. For this model an exact elastodynamic solution exists (Achenbach, 1975), which enables us to solve the problem of energy flow analytically. Moreover, a physical interpretation of elastic near-field wave energy is straightforward and well comprehensible for a spherical cavity and will be shown to be generalized to more complex sources inclusive of an earthquake source.

II. ELASTODYNAMIC FIELD GENERATED BY A SPHERICAL CAVITY

For a perfectly elastic, homogeneous, isotropic, and unbounded medium, the elastodynamic equation is

$$\rho \ddot{u}_i - (\lambda+\mu) u_{j,ji} - \mu u_{i,jj} = f_i \ , \qquad (1)$$

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where $u_i(\mathbf{x},t)$ is the displacement vector, λ and μ are Lame's coefficients describing elastic properties of the isotropic medium, ρ is the density of the medium, and $f_i(\mathbf{x},t)$ is the body force vector. Dots over quantities mean the time derivative; indices after the comma denote the spatial derivative. The Einstein summation convention of pairs of equal indices is applied. If we assume a spherically symmetric point source by

$$f_i=\delta_{in}M(t)\frac{\partial\delta(\mathbf{x})}{\partial x_n} , \tag{2}$$

the exact solution of Eq. (1) can be written as follows (Achenbach, 1975; Aki and Richards, 1980)

$$u_i(\mathbf{x},t)=\frac{\gamma_i}{4\pi\rho\alpha^2}\frac{M(t-r/\alpha)}{r^2}+\frac{\gamma_i}{4\pi\rho\alpha^3}\frac{\dot{M}(t-r/\alpha)}{r} , \tag{3}$$

where γ_i are the direction cosines of the position vector $\mathbf{x}$, r—the magnitude of $\mathbf{x}$—is the distance of an observer from the source, $\alpha=\sqrt{\lambda+2\mu}$ is the P-wave velocity, and $M(t)$ is the source-time function.

The first and second terms in Eq. (3) are called *near-field P waves* (P^N) and *far-field P waves* (P^F), respectively. No S waves are generated by this source. If $M(t)$ is time independent, the elastodynamic equation (equation of motion) reduces to the elastostatic equation (equation of equilibrium), and the solution consists only of the first term in Eq. (3). Accordingly the near-field waves are related to the static field, or more exactly, the near-field waves are responsible for any change of the static field. The amplitudes of the P^N waves decrease as $1/r^2$; the amplitudes of the P^F waves as $1/r$. Both waves are longitudinal with a spherically symmetric radiation pattern.

Equation (3) also describes the displacement field of a spherical cavity with radius ε, if the displacement generated by a point source at distance $r=\varepsilon$ coincides with the displacement at the cavity surface. In this case the source process at the cavity surface is retarded by a time ε/α, and the time dependence of the cavity surface displacement is not identical with source-time function $M(t)$, although it obviously converges to $M(t)$ as $\varepsilon\to 0$. We consider here two forms of $M(t)$ (see Fig. 1) corresponding to expanding and contracting cavities. The *expanding cavity* is a source that expands in response to external forces applied to its surface. After the source process termination (i.e., at $t\geq\varepsilon/\alpha+T$) the external forces hold the cavity in an expanded state. The cavity expansion generates elastic waves that stress the medium about the cavity. A *contracting cavity* is at first in an expanded state and contracts during the source process in response to the ceasing of external forces on its surface. The contraction generates elastic waves and relaxes the stress in the medium surrounding the cavity. If the force is interrupted suddenly at $t=\varepsilon/\alpha$, no further external forces act during the contraction, which is spontaneous. For a *spontaneously contracting cavity,* no external power is supplied to the medium during the source process, thus conserving the total amount of energy in the medium.

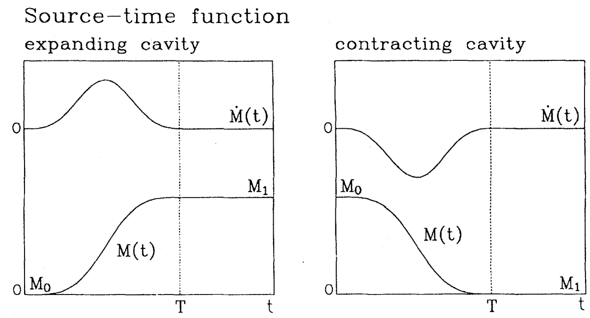

FIG. 1. Source-time function $M(t)$ and its time derivative for expanding and contracting cavities. T denotes the source process duration.

III. DEFINITIONS OF ENERGY AND ENERGY FLUX

The *elastic energy density* $h(\mathbf{x},t)$ and the *energy-flux density* $p_i(\mathbf{x},t)$ can be expressed as follows (Ben-Menahem and Singh, 1981)

$$h(\mathbf{x},t)=h_K+h_U$$
$$=\tfrac{1}{2}\rho\dot{u}_i\dot{u}_i+\tfrac{1}{2}[\lambda u_{i,i}u_{k,k}+\mu u_{i,j}(u_{i,j}+u_{j,i})] , \tag{4}$$
$$p_i(\mathbf{x},t)=-\tau_{ij}\dot{u}_j$$
$$=-\lambda u_{k,k}\dot{u}_i-\mu(u_{i,j}+u_{j,i})\dot{u}_j , \tag{5}$$

where $h_K(\mathbf{x},t)$ and $h_U(\mathbf{x},t)$ are the kinetic and potential (strain) energy densities, and $\tau_{ij}(\mathbf{x},t)$ is the stress tensor. Integrating Eqs. (4) and (5) over the surface of a sphere S with radius r, we introduce the *surface elastic energy* $H(r,t)$, and the *energy flux* $P(r,t)$ flowing out of the sphere S,

$$H(r,t)=\int\int_S h(\mathbf{x},t)dS , \tag{6}$$

$$P(r,t)=\int\int_S p_i(\mathbf{x},t)n_i dS , \tag{7}$$

where n_i is the outer normal of S. Consequently, the *total elastic energy* $T_\varepsilon(t)$ contained in the whole medium outside the cavity and the *wave energy* $W(r)$ flowing out of the sphere S over the whole time history are defined as follows

$$T_\varepsilon(t)=\int_\varepsilon^\infty H(r,t)dr , \tag{8}$$

$$W(r)=\int_0^\infty P(r,t)dt . \tag{9}$$

IV. TOTAL AND WAVE ENERGY

The description (3) of the elastodynamic field generated by a spherical cavity affords expressing all the energy quantities in exact analytical form and thereby the energy balance of the complete wave field *exactly.* These formulas are quite complex, because they consist not only of far-field and near-field energy terms, but also of interaction terms between the far-field and near-field waves. In this section we study only the general energy balance of the wave field. In the next section, individual terms of the energy will be discussed in detail.

A. Total elastic energy $T_\varepsilon(t)$

The total energy $T_\varepsilon(t)$ of the elastodynamic field (3) can be expressed as follows: before the source process begins, i.e., at $t \le \varepsilon/\alpha$,

$$T_{\varepsilon 0} = T_\varepsilon(t \le \varepsilon/\alpha) = \frac{\mu}{4\pi\rho^2\alpha^4}\frac{2M_0^2}{\varepsilon^3}\ , \tag{10}$$

and, after the source process terminates, $t \ge \varepsilon/\alpha + T$,

$$T_{\varepsilon 1} = T_\varepsilon(t \ge \varepsilon/\alpha + T)$$
$$= \frac{1}{4\pi\rho^2\alpha^4}\left\{\mu\frac{2M_1^2}{\varepsilon^3} + \frac{\lambda+2\mu}{\alpha^3}\int_0^T \ddot{M}^2(t)dt\right\}\ . \tag{11}$$

For the *expanding cavity,* the initial value of total elastic energy $T_\varepsilon(t)$ equals zero. It increases during the source process, because the external forces at the cavity surface supply energy to the medium. When the source process terminates ($t = \varepsilon/\alpha + T$), $T_\varepsilon(t)$ reaches the final value, being constant thereafter (see Fig. 2). This final value is the sum of the static energy [first term in Eq. (11)] loaded into the medium, and of the energy of the far-field waves [second term in Eq. (11)] radiated by the cavity. For the *contracting cavity,* the initial total elastic energy $T_{\varepsilon 0}$ equals the static energy (10) previously accumulated. In the final state, $T_\varepsilon(t)$ equals only the elastic energy of the far-field waves [the second term in Eq. (11)], because the medium has become unstrained. The ratio $T_{\varepsilon 0}/T_{\varepsilon 1}$ depends on the form of source-time function $M(t)$. For a high-frequency range, $T_{\varepsilon 1}$ is greater than $T_{\varepsilon 0}$ because of dominance of the $\int \ddot{M}^2(t)dt$ term, and vice versa for a low-frequency range. For $M(t)$ to be of high frequency, the forces at the cavity surface must accelerate the contraction, supplying energy to the medium. On the contrary, the low-frequency $M(t)$ is caused by the deceleration of the source process by forces opposing the contraction. The cavity surface then expends energy during contraction, decreasing the total energy of the medium. For the *spontaneously contacting cavity,* no external forces act during the contraction, so that $T_{\varepsilon 0} = T_{\varepsilon 1}$ (Fig. 2).

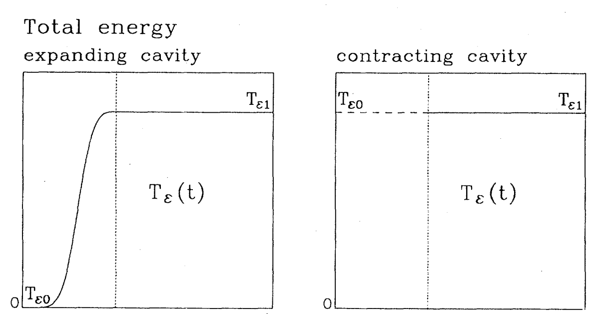

FIG. 2. Total energy T_ε as a function of time. The dashed line for times $t < \varepsilon/\alpha + T$ (right-hand plot) displays $T_\varepsilon(t)$ during the source process for a spontaneously contracting cavity. The vertical dotted line marks the end of the source process.

B. Wave energy $W(r)$

Regarding the wave energy $W(r)$, the formula (3) yields

$$W(r) = \frac{1}{4\pi\rho^2\alpha^4}\left\{2\mu\frac{M_1^2 - M_0^2}{r^3} + \frac{\lambda+2\mu}{\alpha^3}\int_0^T \ddot{M}^2(t)dt\right\}\ . \tag{12}$$

The first and second terms of Eq. (12) correspond to the wave energies of the near-field and far-field waves, respectively. The near-field wave energy can be either positive or negative and depends on the distance, vanishing at large distances. At the cavity surface it has exactly the same value as the static energy [see Eqs. (10) and (11)], which is loaded into the medium (expanding cavity) or released (contracting cavity) during the wave propagation through the medium. In contrast, the far-field wave energy is always positive and does not depend on the distance. For an *expanding cavity,* $W(r = \varepsilon)$ equals the work spent by external forces at the cavity surface. $W(r)$ decreases as the distance increases, because the near-field wave energy transforms into static energy. At larger distances, $W(r)$ converges to the far-field wave energy. For the *contracting cavity,* the wave energy $W(r)$ is an increasing function of distance, also converging to the far-field wave energy. The increase of $W(r)$ results from the static-to-wave energy transformation. The value of the wave energy at the cavity surface depends on the external forces acting at the surface. If the contraction of the cavity is accelerated, a positive energy is produced by the forces, resulting in positive values of W at $r = \varepsilon$. If the contraction decelerates, the cavity surface expends work, and W becomes negative at $r = \varepsilon$. For the *spontaneously contracting cavity,* no external forces act at its surface, and no wave energy is radiated. The zero value of W at $r = \varepsilon$ can also be obtained directly from formula (5), if we take into account that the surface of the spontaneously contracting cavity is by definition free of traction.

For the decelerated contracting cavity, the wave energy at the cavity surface and in its vicinity can be *negative.* It should be emphasized that negative values of the wave energy represent not only a formal consequence of a definition of the direction of wave propagation, but have a well-founded physical interpretation. A negative energy means that energy is transported by waves in the direction opposite to that of wave propagation. Consequently, the wave radiated by the cavity serves to transport energy from the medium to the cavity. In other words, the cavity radiates elastic waves into the medium, but radiates no positive wave energy, since energy is absorbed by the cavity surface. This statement seems to clash with our intuition, but does not actually disturb any physical principle. Negative values of elastic energy density $h(\mathbf{x},t)$ would be physically unacceptable, but a negative wave energy does not imply that.

V. TYPES OF ENERGY AND ENERGY FLUX

For the elastodynamic field generated by a spherical cavity (3), the surface energy $H(r,t)$ and the energy flux $P(r,t)$ are given by

$$H(r,t)=\frac{\mu}{4\pi\rho^2}\left\{\frac{6}{\alpha^4}\frac{M^2(t-r/\alpha)}{r^4}+\frac{12}{\alpha^5}\frac{M(t-r/\alpha)\dot{M}(t-r/\alpha)}{r^3}+\frac{4}{\alpha^6}\frac{M(t-r/\alpha)\ddot{M}(t-r/\alpha)}{r^2}+\frac{6}{\alpha^6}\frac{\dot{M}^2(t-r/\alpha)}{r^2}+\frac{4}{\alpha^7}\frac{\dot{M}(t-r/\alpha)\ddot{M}(t-r/\alpha)}{r}\right\}+\frac{\lambda+2\mu}{4\pi\rho^2}\left\{\frac{1}{2\alpha^6}\frac{\dot{M}^2(t-r/\alpha)}{r^2}+\frac{1}{\alpha^7}\frac{\dot{M}(t-r/\alpha)\ddot{M}(t-r/\alpha)}{r}+\frac{1}{\alpha^8}\ddot{M}^2(t-r/\alpha)\right\}, \tag{13}$$

$$P(r,t)=\frac{\mu}{4\pi\rho^2}\left\{\frac{4}{\alpha^4}\frac{M(t-r/\alpha)\dot{M}(t-r/\alpha)}{r^3}+\frac{4}{\alpha^5}\frac{M(t-r/\alpha)\ddot{M}(t-r/\alpha)}{r^2}+\frac{4}{\alpha^5}\frac{\dot{M}^2(t-r/\alpha)}{r^2}+\frac{4}{\alpha^6}\frac{\dot{M}(t-r/\alpha)\ddot{M}(t-r/\alpha)}{r}\right\}+\frac{\lambda+2\mu}{4\pi\rho^2}\left\{\frac{1}{\alpha^6}\frac{\dot{M}(t-r/\alpha)\ddot{M}(t-r/\alpha)}{r}+\frac{1}{\alpha^7}\ddot{M}^2(t-r/\alpha)\right\}. \tag{14}$$

In order to understand the time-space evolution of energy and energy flux, we split the relations (13) and (14) into several terms, representing physically different forms of energy and energy flux:

$$H=H^S+H^D+H^T+H^R\ , \tag{15}$$

$$P=P^D+P^T+P^R\ , \tag{16}$$

calling H^S, H^D, H^T, and H^R the static, dynamic, transient, and residual parts of H, and P^D, P^T, and P^R the dynamic, transient, and residual parts of P, respectively.

A. Static term

Static energy H^S is the part of H that is not concentrated in the time-space interval of elastic waves only $[\alpha(t-T)\le r\le\alpha t]$, but can be distributed through the whole medium:

$$H^S=\frac{6\mu}{4\pi\rho^2\alpha^4}\frac{M^2(t-r/\alpha)}{r^4}\ . \tag{17}$$

H^S is concentrated mainly near the source, decreasing rapidly with the distance [Fig. 3(a)]. Every loading or release of static energy is performed by near-field waves, implying that any change of static energy is gradual and in accordance with the causality principle. No flux term or wave-energy term corresponds to the static energy.

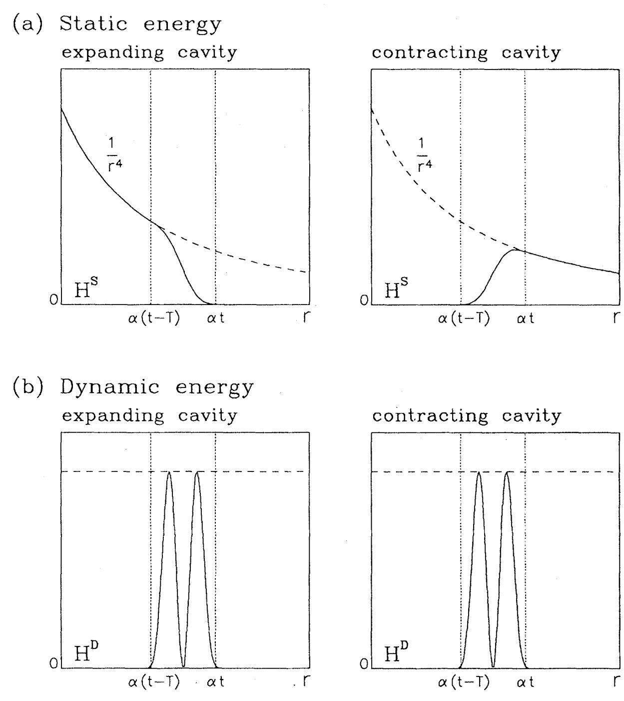

FIG. 3. Surface energy as a function of distance at a fixed instant t: (a) static surface energy H^S; (b) dynamic surface energy H^D. The vertical dotted lines delineate the space interval of the P wave. The dashed line shows the envelope of H^S and H^D for different times.

B. Dynamic term

The *dynamic energy* H^D and the *dynamic energy flux* P^D are parts of H and P that do not depend on the distance from the source explicitly:

$$H^D = \frac{\lambda+2\mu}{4\pi\rho^2\alpha^8}\ddot{M}^2(t-r/\alpha) , \tag{18}$$

$$P^D = \frac{\lambda+2\mu}{4\pi\rho^2\alpha^7}\ddot{M}^2(t-r/\alpha) . \tag{19}$$

H^D is the local energy appearing only in the time-space interval of elastic waves and always positive [Fig. 3(b)]. The *dynamic total energy* T_ε^D and the *dynamic wave energy* W^D are expressed as follows:

$$T_\varepsilon^D(t) = \frac{\lambda+2\mu}{4\pi\rho^2\alpha^8}\int_\varepsilon^\infty \ddot{M}^2(t-r/\alpha)dr , \tag{20}$$

$$W^D = \frac{\lambda+2\mu}{4\pi\rho^2\alpha^7}\int_0^T \ddot{M}^2(t)dt . \tag{21}$$

The initial value of T_ε^D vanishes, but it increases during the source process, becoming constant after its termination. The wave energy W^D is constant at all distances from the source, and corresponds to the conventionally determined *radiated energy* for $r \to \infty$.

C. Transient term

The *transient energy* H^T is a part of the remnant energy H-H^S-H^D that contributes by a nonzero amount to the total energy $T_\varepsilon(t)$ after the source process has ended; the *transient energy flux* P^T is part of P-P^D that contributes to the wave energy $W(r)$:

$$H^T = \frac{4\mu}{4\pi\rho^2\alpha^5}\frac{M(t-r/\alpha)\dot{M}(t-r/\alpha)}{r^3} , \tag{22}$$

$$P^T = \frac{4\mu}{4\pi\rho^2\alpha^4}\frac{M(t-r/\alpha)\dot{M}(t-r/\alpha)}{r^3} . \tag{23}$$

H^T is the local energy [Fig. 4(a)], which can be either positive or negative, as is also P^T. At times $t \geq \varepsilon/\alpha + T$ we have

$$W^T = \frac{2\mu}{4\pi\rho^2\alpha^4}\frac{M_1^2 - M_0^2}{r^3} , \tag{24}$$

$$T_\varepsilon^T(t) + T_\varepsilon^S(t) = T_{\varepsilon 1}^S = \frac{2\mu}{4\pi\rho^2\alpha^4}\frac{M_1^2}{\varepsilon^3} , \tag{25}$$

where W^T denotes the *transient wave energy,* T_ε^S and T_ε^T the *total static* and *transient energies,* and $T_{\varepsilon 1}^S = T_\varepsilon^S(t \to \infty)$ the final total static energy.

The static and transient energies form the closed energy system (25). The sum of these energies is constant, but each of these energies can transform into one another in the course of time. For the *expanding cavity,* $T_\varepsilon^T(t)$ stems from external forces during the source process. Once the source process has come to an end, no external power is supplied, and $T_\varepsilon^T(t)$ decreases, transforming into static energy until it vanishes [Fig. 5(a)]. Likewise, $W^T(r)$ is a decreasing function converging to zero as $r \to \infty$ [Fig. 5(b)]. The decrease of $W^T(r)$ results from the wave-to-static energy transformation during the loading of static energy into the medium. For the *contracting cavity,* $T_\varepsilon^T(t)$ is negative immediately after the source process has ended. The release of static energy reduces this energy deficit, which vanishes when all the static energy has been released [Fig. 5(a)]. For the *spontaneously contracting cavity,* the wave energy $W^T(r)$ is a negative and increasing function of distance [Fig. 5(b)]. This increase of $W^T(r)$ reflects the transformation of static into transient wave energy. The transient energy flux $P^T(r)$ is directed towards the source, whereby the waves transmit energy in a direction opposite to the wave propagation. $W^T(r)$ and $P^T(r)$ vanish once all static energy has been released.

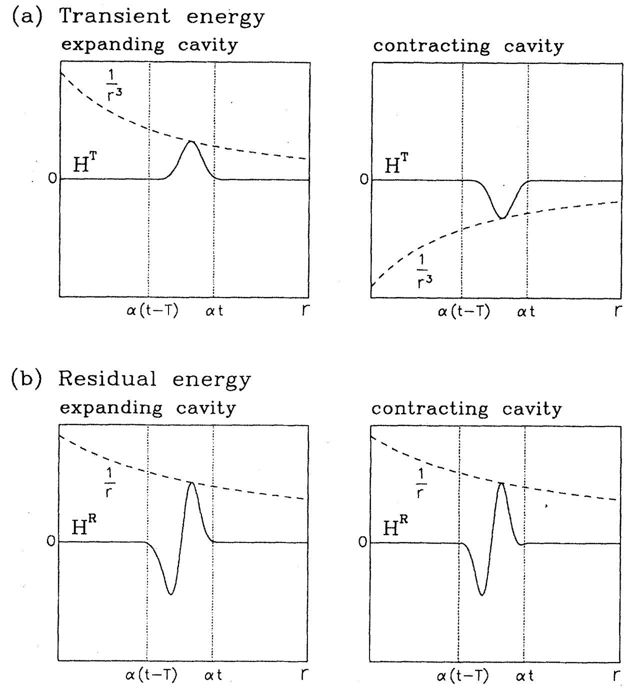

FIG. 4. Surface energy as a function of distance at a fixed instant t: (a) transient surface energy H^T; (b) residual surface energy H^R. The vertical dotted lines delineate the space interval of the P wave. The dashed line shows the envelope of maximum amplitudes of H^T and H^R at different times.

D. Residual term

Residual energy H^R [Fig. 4(b)] and *residual energy flux* P^R are defined as residues of the surface energy H and energy flux P. The *total residual energy* T_ε^R and *residual wave energy* W^R vanish identically, implying that the residual term does not affect the total energy balance, only redistributing energy in the elastic waves. It is noteworthy, however, that the relatively slow decrease of H^R with distance (as $1/r$) allows it to affect the time-

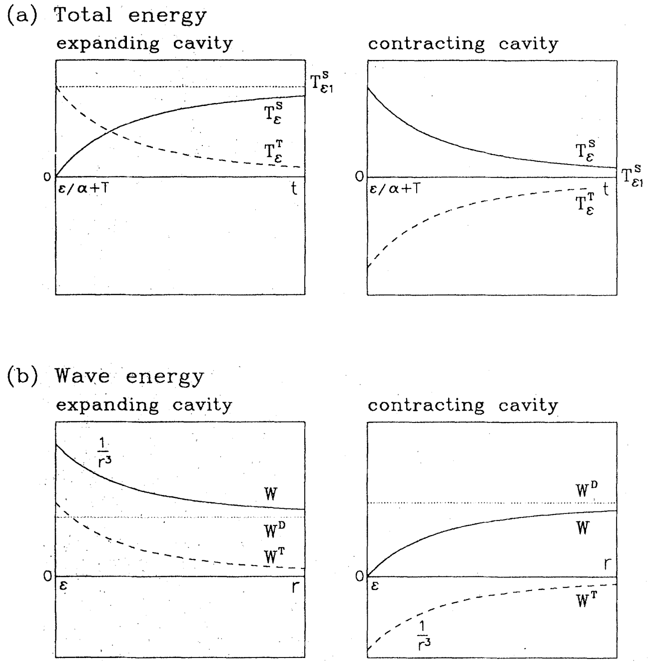

FIG. 5. Schematic energy plots: (a) Transient total energy T_ε^T (dashed line) and static total energy T_ε^S (solid line) as a function of time for an expanding and spontaneously contracting cavity. The functions are represented after the end of the source process (time $t \geq \varepsilon/\alpha + T$). $T_{\varepsilon 1}^S$ denotes the limit of $T_\varepsilon^S(t)$ as $t \rightarrow \infty$. (b) Solid line, wave energy W; dashed line, transient wave energy W^T; dotted line, dynamic wave energy W^D, all as functions of distance.

space form of $H(r,t)$ considerably at the distances where H^S and H^T practically vanish.

VI. CONCLUSIONS

Our study of the energy balance of the elastodynamic field generated by a spherical cavity shows that a proper understanding requires taking into account the *energy of the near-field waves.* The presence of the near-field wave energy is necessary to perform the static-to-wave and wave-to-static energy transformations.

For an *expanding cavity,* the work carried out by the external forces acting at the cavity surface is stored in the elastic waves immediately after the end of the source process. The wave energy peaks at the cavity surface, decreasing monotonically with increasing distance and then converging to the energy of the far-field waves. The decrease in the wave energy results from the wave-to-static energy transformation. Loading of the static energy into the medium is performed gradually by the near-field waves. For a *contracting cavity,* all the energy is stored in the form of static energy prior to the source process. This energy is contained in the medium surrounding the cavity rather than concentrated on the cavity surface. During its contraction, the cavity surface generates elastic waves without requiring additional radiation of positive wave energy. In a *spontaneously contracting cavity,* its surface radiates no wave energy; in the case of decelerated contraction, the cavity surface even dissipates energy during the wave radiation. As the radiated waves propagate from the cavity through the medium, they release the static energy stored there, transforming it into wave energy. Thus the wave energy increases, converging to the energy of the far-field waves. Simply, *for an expanding cavity the elastic near-field waves transport and distribute the static energy into the medium, so that the wave energy decreases, while, for a contracting cavity, the elastic near-field waves propagating through a stressed medium release the static energy from the medium, transforming it into increasing wave energy.*

The energy balance in earthquakes is further complicated by the presence of a rupture process, by complexities of the earthquake source geometry, by inhomogeneities in the source and in the medium, by absorption and anisotropy of the medium, etc., but the general principle of energy balance remains nevertheless the same. In analogy to the cavity with decelerated contraction, the surface of a real earthquake source dissipates energy to crack growth and to friction at the fault, leading to negative values of the wave energy at and near the fault. The negative wave energy serves to transport energy from the medium to the fault. After release of the static strain energy by seismic near-field waves, the wave energy becomes positive, converging to the far-field wave energy at large distances.

Apparently, a quite analogous approach can be applied not only in seismology, but to all wave-field problems with different initial and final distributions of static energy. In these cases the static energy shares in the energy balance, originating from the wave energy or transforming into it. This transformation does not take place at a single instant for the whole field, but is effected gradually *by the near-field waves* in accordance with the causality principle. The near-field waves, moreover, have an exceptional and amazing property, being able to *transmit wave energy towards the source,* i.e., in the direction opposite to wave propagation. This strange conclusion reveals the incorrectness of the notion of identifying the source of waves with the source of energy. In fact, during the process of wave generation, the energy can even be dissipated by the wave source.

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REFERENCES

Achenbach, J. D., 1975, *Wave Propagation in Elastic Solids* (North-Holland, Amsterdam).

Aki, K., and P. G. Richards, 1980, *Quantitative Seismology, Theory and Methods I* (Freeman, San Francisco).

Ben-Menahem, A., and S. J. Singh, 1981, *Seismic Waves and Sources* (Springer-Verlag, Heidelberg).
Červený, V., I. A. Molotkov, and I. Pšenčík, 1977, *Ray Method in Seismology* (Charles University, Prague).
Freund, L. B., 1990, *Dynamic Fracture Mechanics* (Cambridge University, Cambridge, England).
Kostrov, B. V., and S. Das, 1989, *Principles of Earthquake Source Mechanics* (Cambridge University, Cambridge, England).
Madariaga, R., 1986, in *Earthquakes: Observation, Theory and Interpretation,* edited by H. Kanamori and E. Boschi (North-Holland, Amsterdam), p. 1.
Rudnicki, J. W., 1983, Bull. Seismol. Soc. Am. **73**, 901.
Rudnicki, J. W., and L. B. Freund, 1981, Bull. Seismol. Soc. Am. **71**, 583.
Turcotte, D. L., and G. Schubert, 1982, *Geodynamics. Applications of Continuum Physics to Geological Problems* (Wiley, New York).