3D Anisotropic tomography: A novel method and its application in SW Iceland

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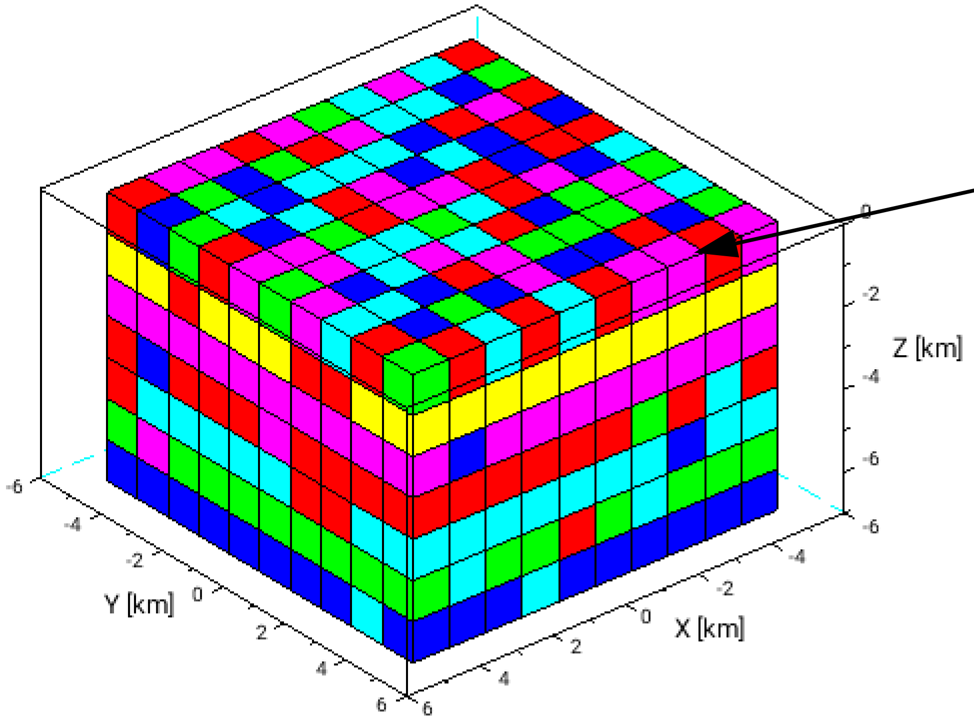
Parameterization of the model



In each box:
α_0, β_0 (background velocities) +
21 anisotropy parameters

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23 parameters in total but,
21 parameters are independent

Following parameterizations are equivalent:

- [α_0, β_0, anisotropy parameters]
- 6 x 6 Voigt matrix

Anisotropy parameters

$$\epsilon_x = \frac{A_{11} - \alpha_0^2}{2\alpha_0^2},\ \ \epsilon_y = \frac{A_{22} - \alpha_0^2}{2\alpha_0^2},\ \ \epsilon_z = \frac{A_{33} - \alpha_0^2}{2\alpha_0^2},$$

$$\chi_x = \frac{A_{14} + 2A_{56}}{\alpha_0^2},\ \ \chi_y = \frac{A_{25} + 2A_{46}}{\alpha_0^2},\ \ \chi_z = \frac{A_{36} + 2A_{45}}{\alpha_0^2},$$

$$\eta_x = \frac{2(A_{23} + 2A_{44}) - A_{22} - A_{33}}{2\alpha_0^2},\ \ \eta_y = \frac{2(A_{13} + 2A_{55}) - A_{33} - A_{11}}{2\alpha_0^2},\ \ \eta_z = \frac{2(A_{12} + 2A_{66}) - A_{11} - A_{22}}{2\alpha_0^2},$$

$$\xi_{15} = \frac{A_{25} + 2A_{46} - A_{15}}{\alpha_0^2},\ \ \xi_{16} = \frac{A_{36} + 2A_{45} - A_{16}}{\alpha_0^2},\ \ \xi_{24} = \frac{A_{14} + 2A_{56} - A_{24}}{\alpha_0^2},$$

$$\xi_{26} = \frac{A_{36} + 2A_{45} - A_{26}}{\alpha_0^2},\ \ \xi_{34} = \frac{A_{14} + 2A_{56} - A_{34}}{\alpha_0^2},\ \ \xi_{35} = \frac{A_{25} + 2A_{46} - A_{35}}{\alpha_0^2},$$

$$\gamma_x = \frac{A_{44} - \beta^2}{2\beta_0^2},\ \ \gamma_y = \frac{A_{55} - \beta^2}{2\beta_0^2},\ \ \gamma_z = \frac{A_{66} - \beta^2}{2\beta_0^2},\ \ \epsilon_{45} = \frac{A_{45}}{\beta_0^2},\ \ \epsilon_{46} = \frac{A_{46}}{\beta_0^2},\ \ \epsilon_{56} = \frac{A_{56}}{\beta_0^2}$$

P- and common S- waves velocity formula

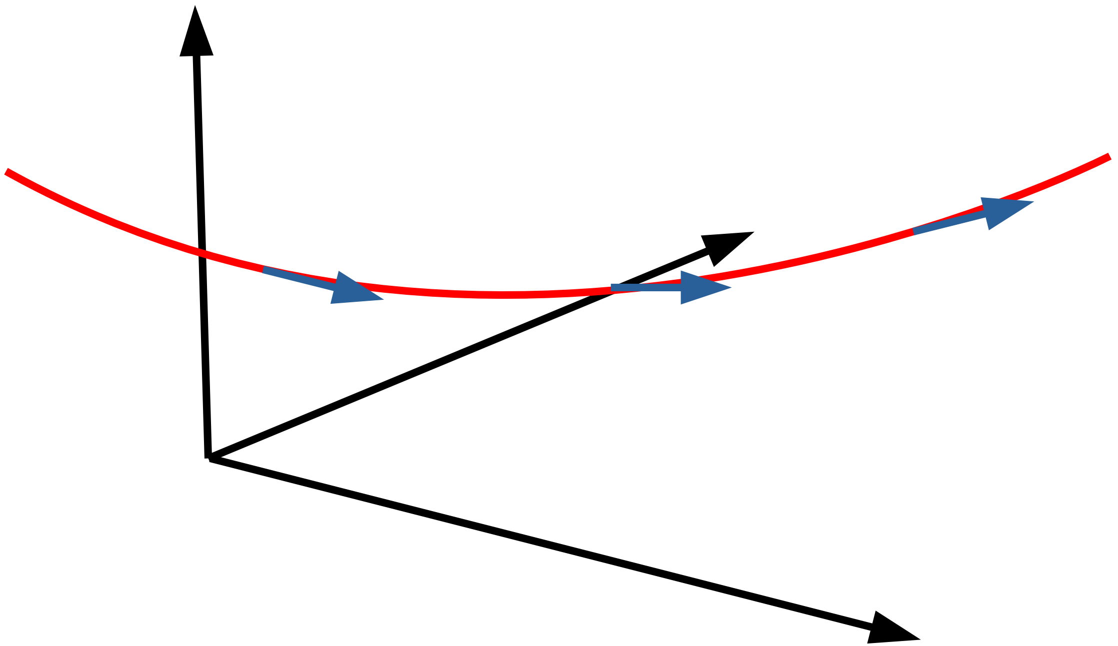
$$\alpha^2 = \alpha_0^2 (1 + 2(n_1^2 \epsilon_x + n_2^2 \epsilon_y + n_3^2 \epsilon_z + n_2^2 n_3^2 \eta_x + n_1^2 n_3^2 \eta_y + n_1^2 n_2^2 \eta_z) + 4 n_2 n_3 (\chi_x - \xi_{24} n_2^2 - \xi_{34} n_3^2) + 4 n_3 n_1 (\chi_y - \xi_{35} n_3^2 - \xi_{15} n_1^2) + 4 n_1 n_2 (\chi_z - \xi_{16} n_1^2 - \xi_{26} n_2^2))$$

$$\beta^2 = \beta_0^2 \{1 + (n_2^2 + n_3^2) \gamma_x + (n_1^2 + n_3^2) \gamma_y + (n_1^2 + n_2^2) \gamma_z + n_1 n_2 \epsilon_{45} + n_1 n_3 \epsilon_{46} + n_2 n_3 \epsilon_{56}\} - \alpha_0^2 (n_2^2 n_3^2 \eta_x + n_1^2 n_3^2 \eta_y + n_1^2 n_2^2 \eta_z + 2 n_1 n_3 (1 - 2 n_1^2) \xi_{15} + 2 n_1 n_2 (1 - 2 n_1^2) \xi_{16} + 2 n_2 n_3 (1 - 2 n_2^2) \xi_{24} + 2 n_1 n_2 (1 - 2 n_2^2) \xi_{26} + 2 n_2 n_3 (1 - 2 n_3^2) \xi_{34} + 2 n_1 n_3 (1 - 2 n_3^2) \xi_{35})$$

α_0, β_0 … background (=reference) velocities

$\boldsymbol{n} = (n_1, n_2, n_3)$ … unit vector in the ray-propagation direction

P- and common S- waves velocity formula



α_0, β_0 … background (=reference) velocities

n = (n_1, n_2, n_3) … unit vector in the ray-propagation direction

Velocity/slowness formula

$$s_P = \frac{1}{v_p} \approx \frac{1}{\alpha_0}\left(1 - \frac{\boldsymbol{g}^P \boldsymbol{m}}{2}\right)$$

$\boldsymbol{g}^p$ ... 21-dim vector constructed from components of unit ray-direction vector

$\boldsymbol{m}$ ... 21-dim vector of anisotropy parameters

Tomographic equations

Matrix constructed from ***g***'s

Anisotropy parameters

Observed traveltime

Traveltime in isotropic background

...

$$\boldsymbol{G}\boldsymbol{m} = \boldsymbol{d}$$

$$\boldsymbol{d} = \boldsymbol{L}\boldsymbol{s} - \boldsymbol{T}^{obs}$$

Velocity/slowness formula

Formula for common S-wave is similar

Tomographic equations

Matrix constructed from ***g***'s

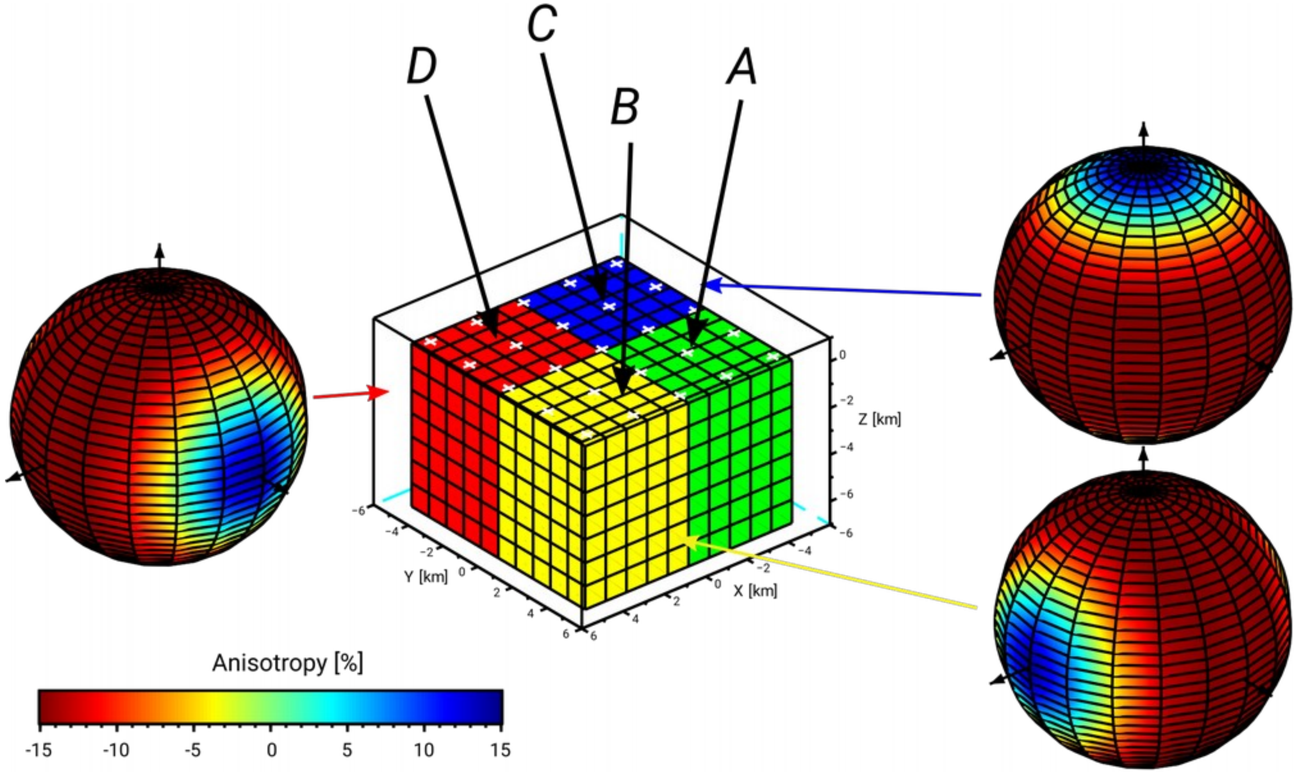
Anisotropy parameters

$$\ldots$$
$$\boldsymbol{G}\,\boldsymbol{m} = \boldsymbol{d}$$
$$\boldsymbol{d} = \boldsymbol{L}\boldsymbol{s} - \boldsymbol{T}^{obs}$$

Observed traveltime

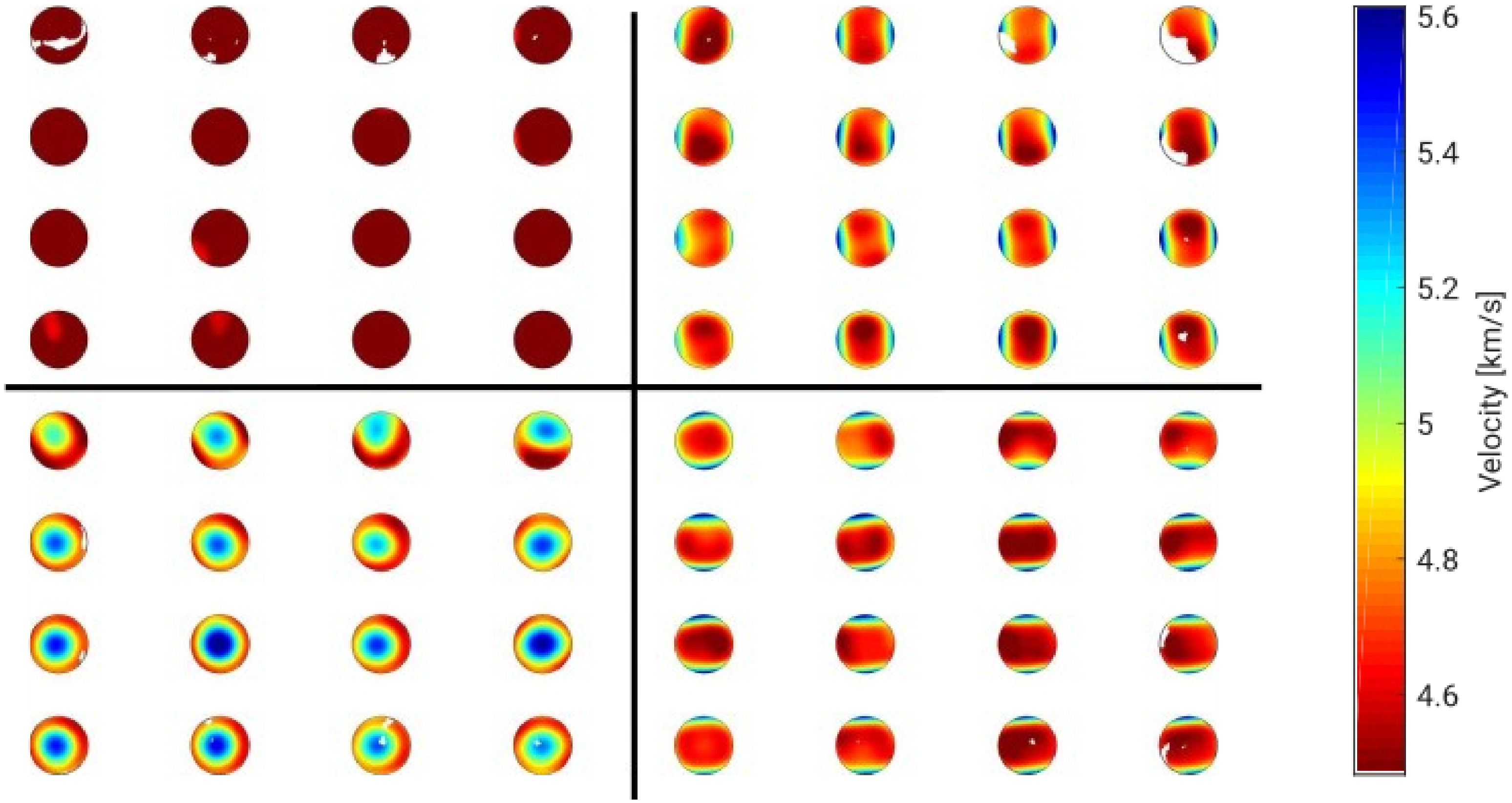
Traveltime in isotropic background

Synthetic example

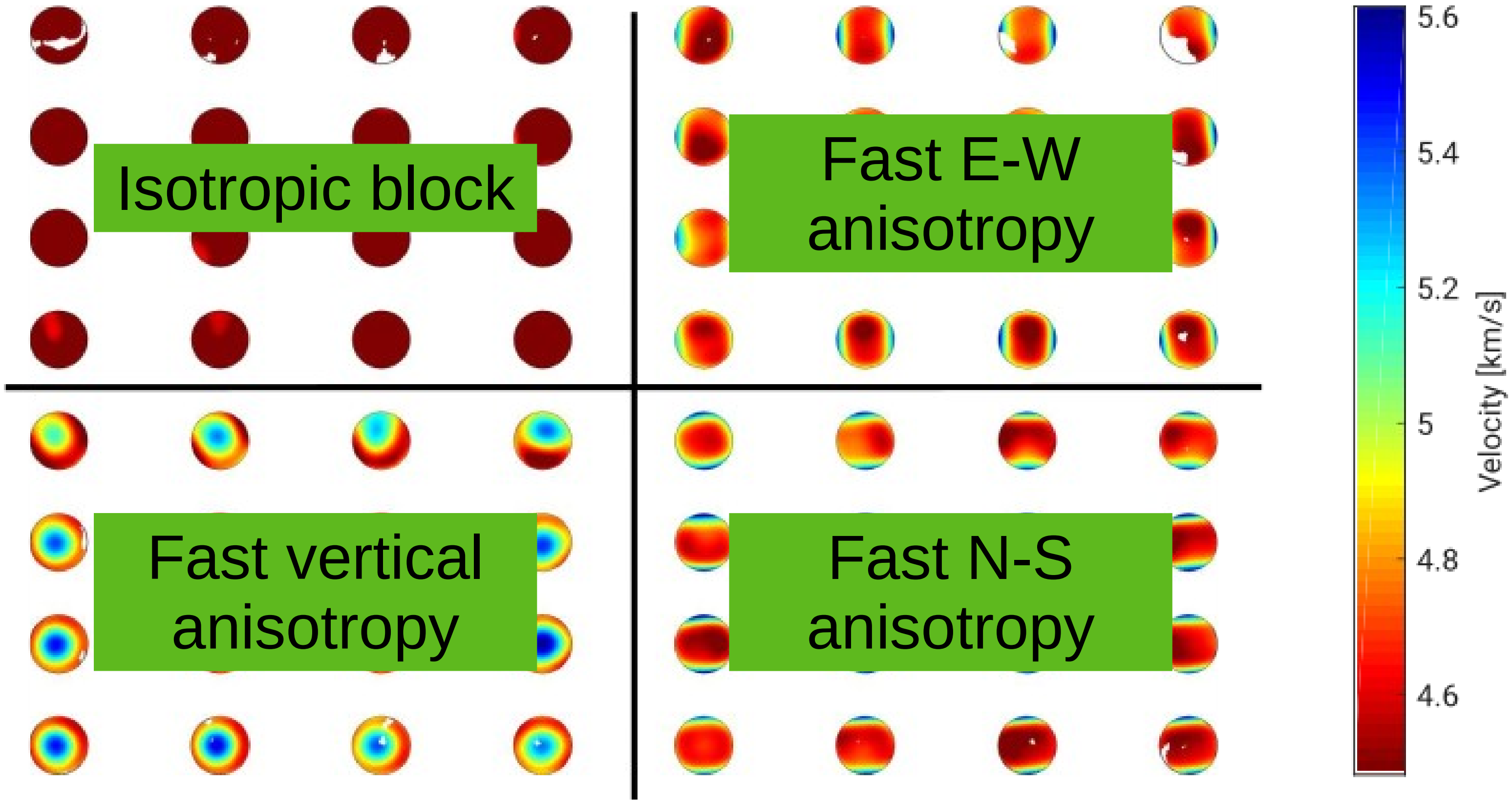


- Hexagonal ±15% anisotropy (B,C,D)
- Green volume isotropic (A)
- Different anisotropy orientation
- Linear vertical gradient of the velocity background

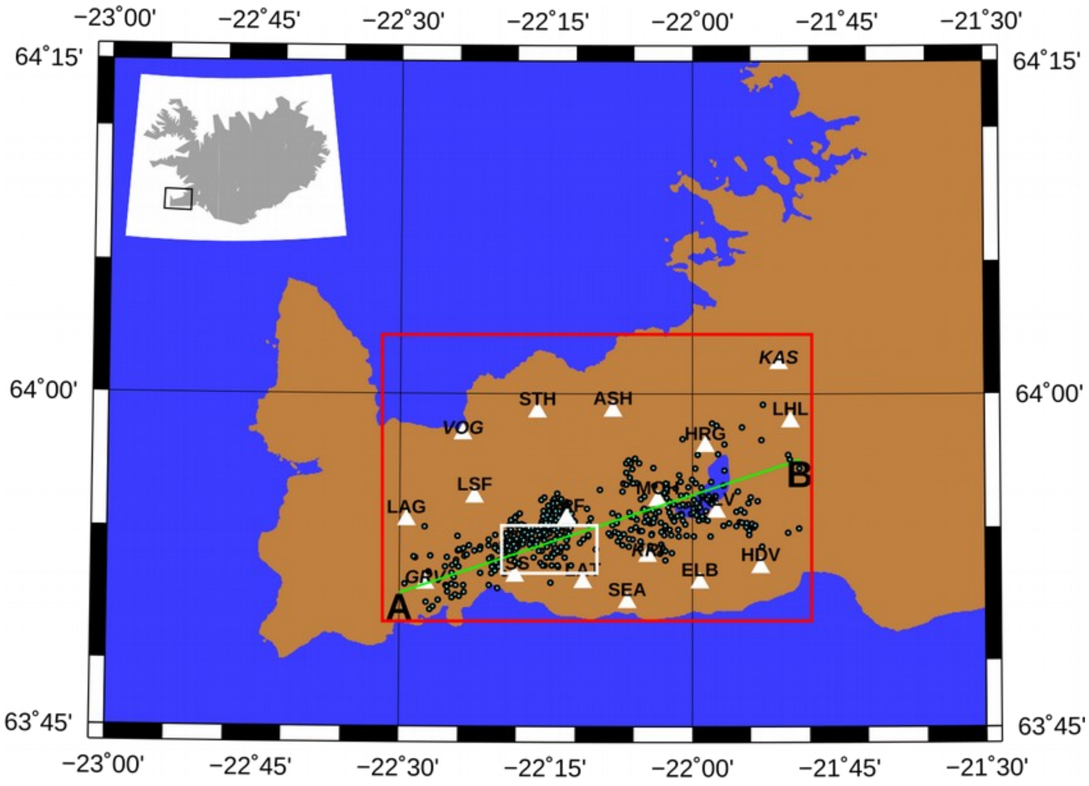
Synthetic example
Inverted P-wave velocity in depth 1.5 km



Synthetic example
Inverted P-wave velocity in depth 1.5 km



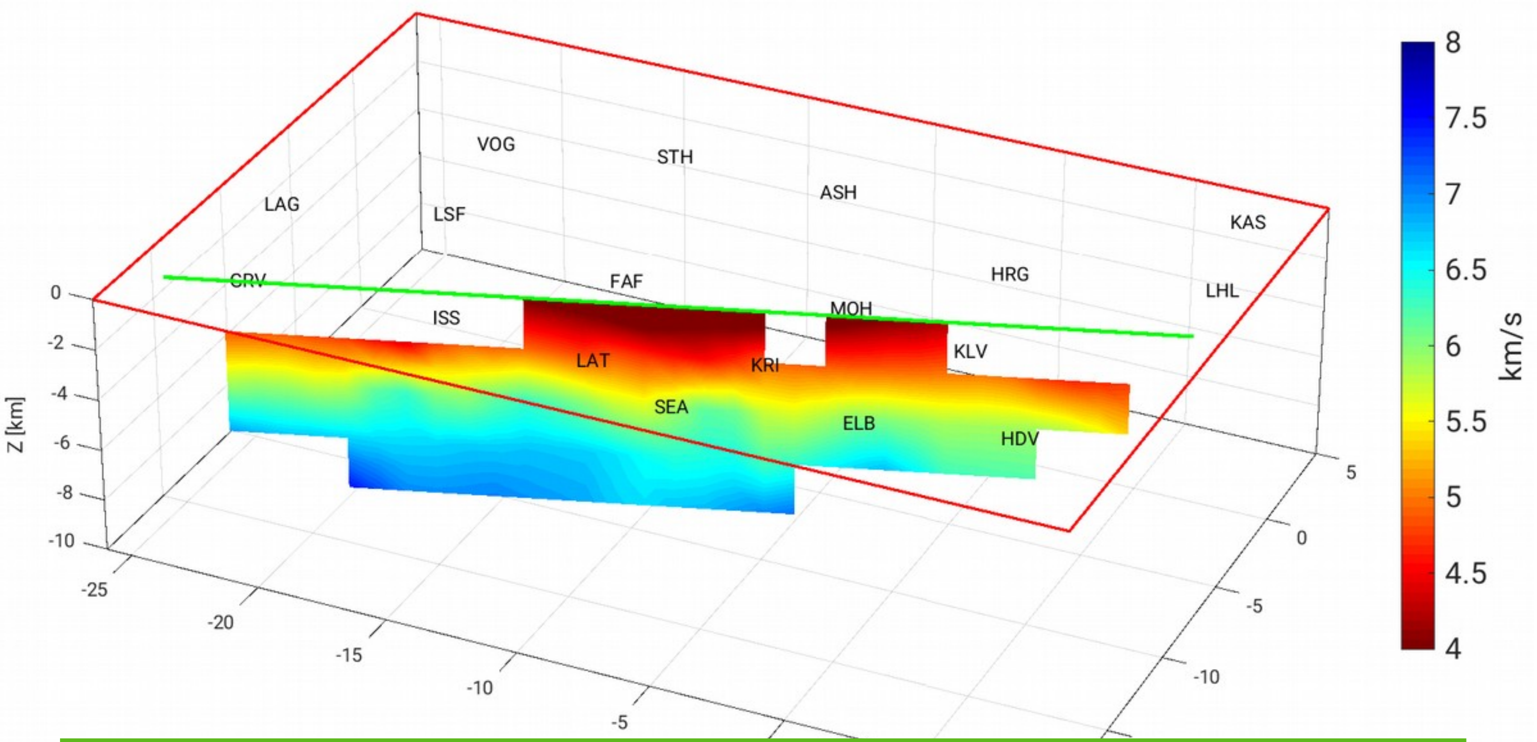
Real inverse problem

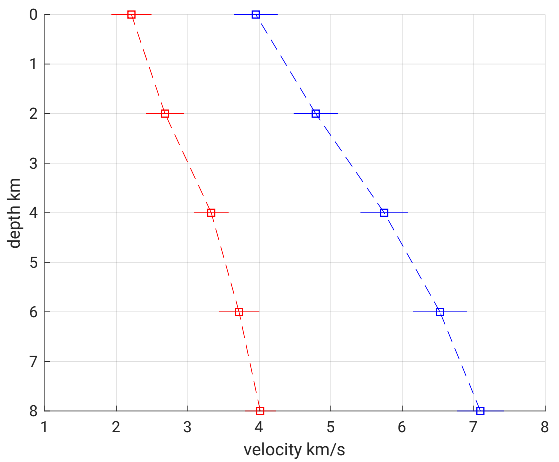


Nr. of boxes: 10 x 10 x 8
Box size km: 2 x 2 x 2
Nr. of illuminated boxes: 566
Nr. of parameters*: **11886**
Nr. of equations*: **8227**
Rank of the system*: **6301**
Min/max/mean ray length km: 2/38/12
Min/max/mean hit count of illuminated boxes: 1/1752/149
Nr. of stations/events: 18/440

*** … critical values that most people fear**

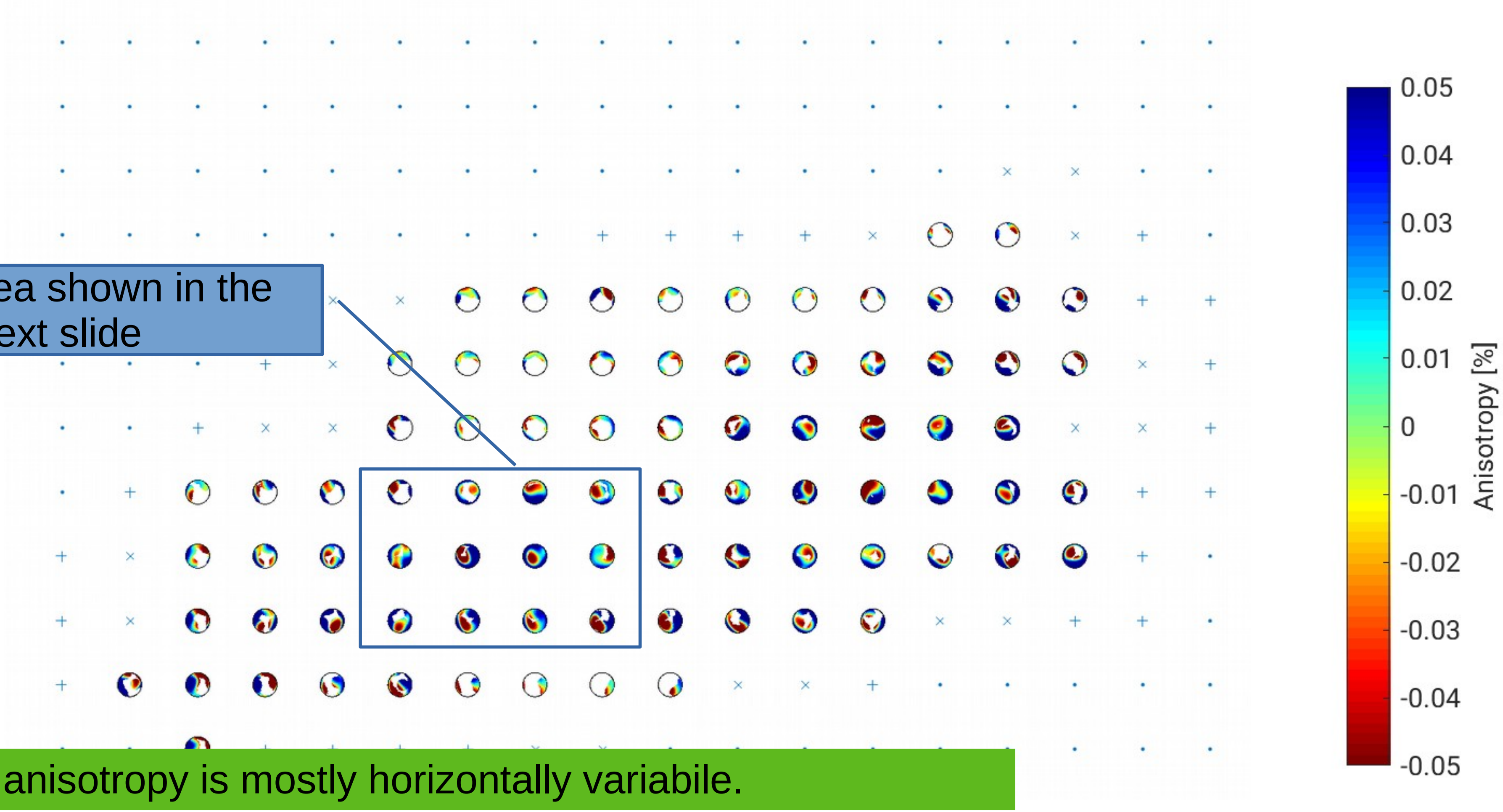
Inversion results – isotropic background





Isotropic background is close to 1D velocity model with low horizontal variability ...

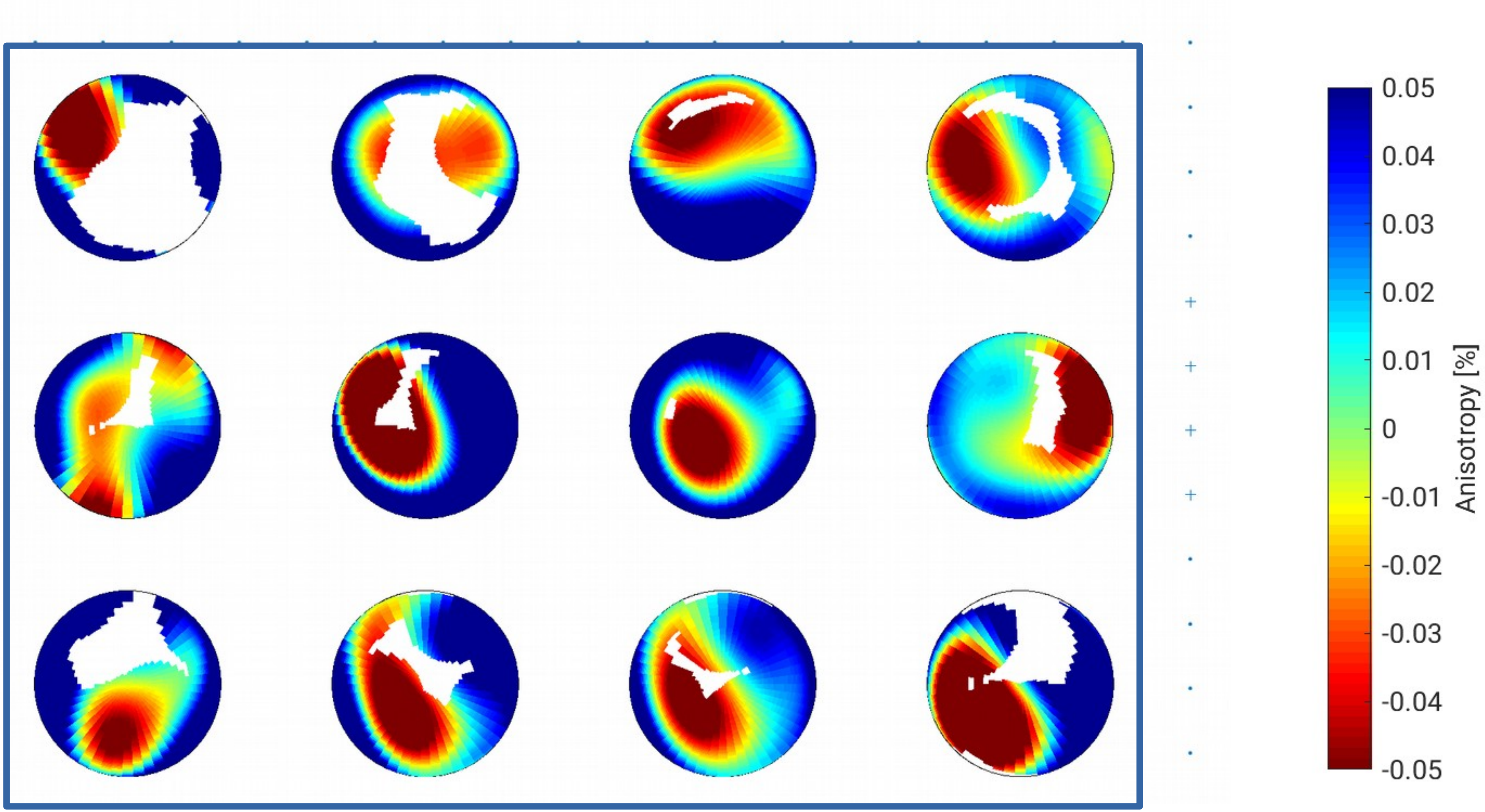
Inversion results – P wave anisotropy 4 km depth



Zoom area shown in the next slide

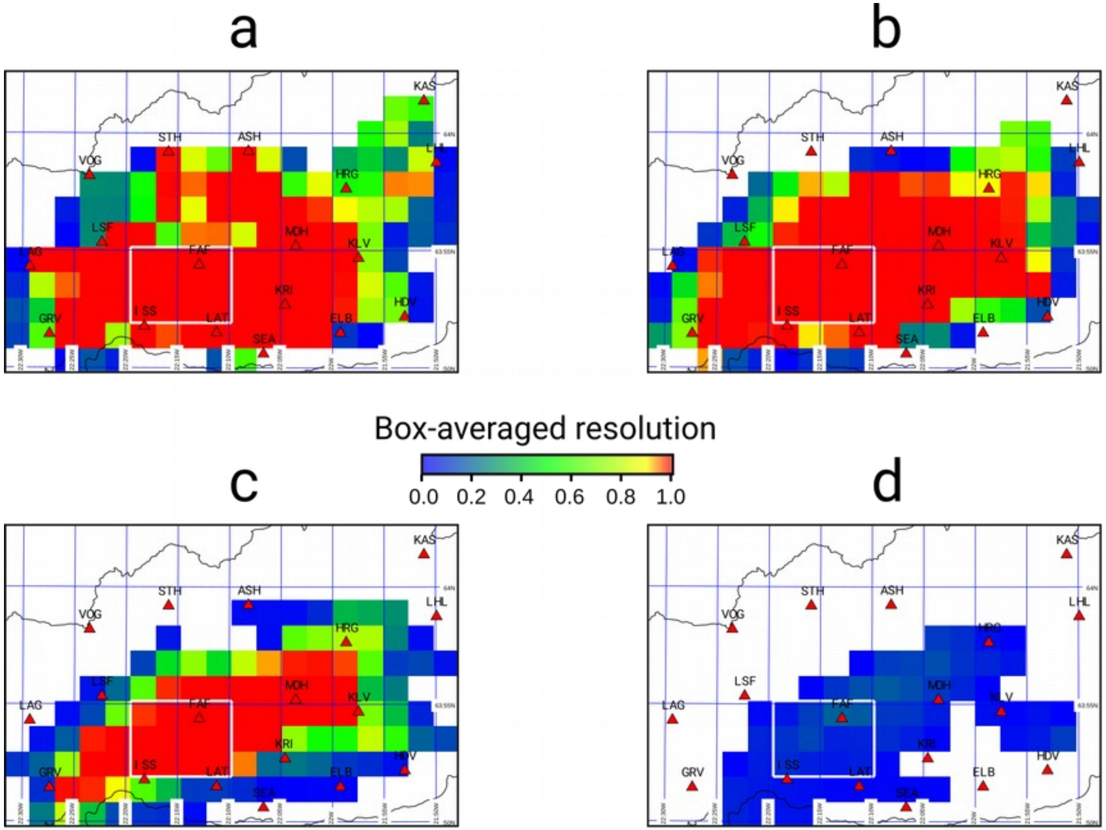
… while anisotropy is mostly horizontally variabile.

Inversion results – P wave anisotropy 4 km depth



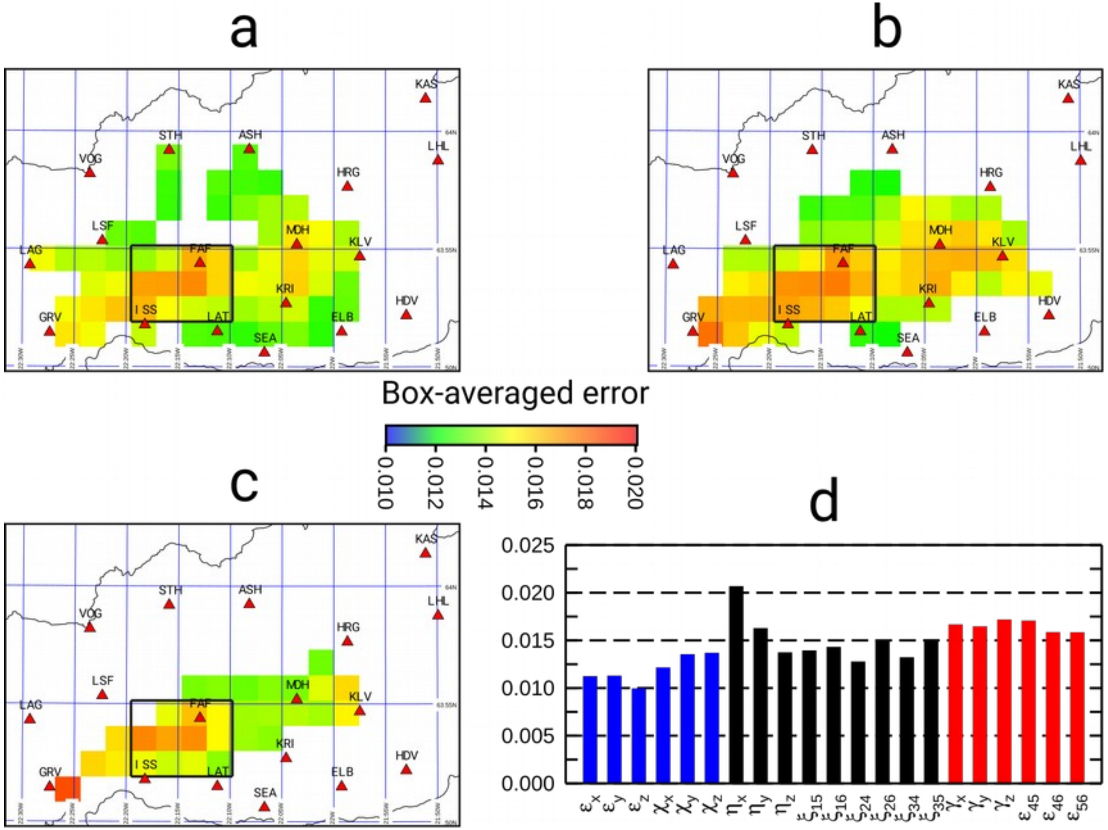
Note similar anisotropy pattern of some neighboring spheres

Resolution at different depth levels (a-d = 2-8 km)



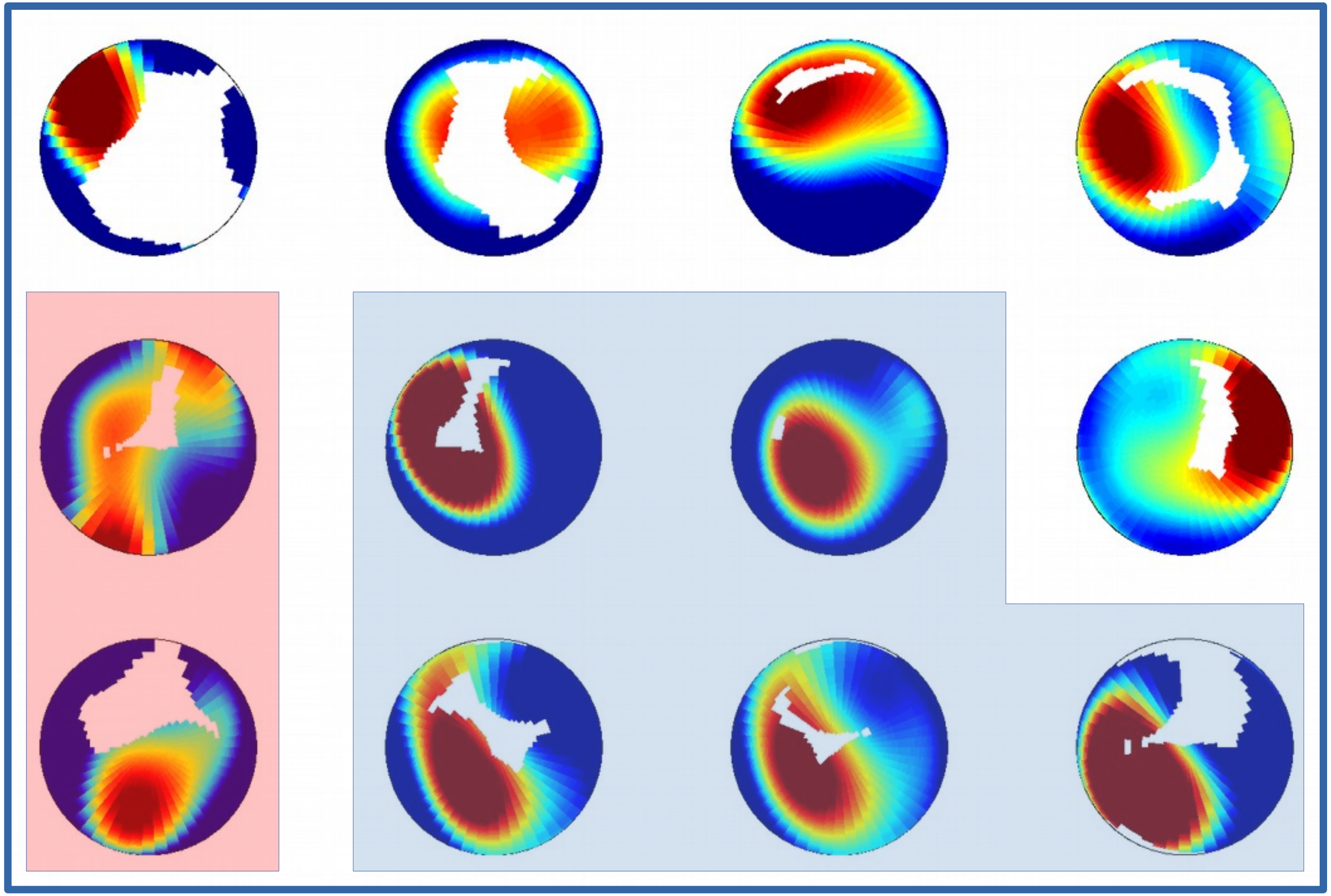
Anisotropy in red boxes is well resolved and ...

Accuracy of anisotropic parameters at different depth levels 2-6 km

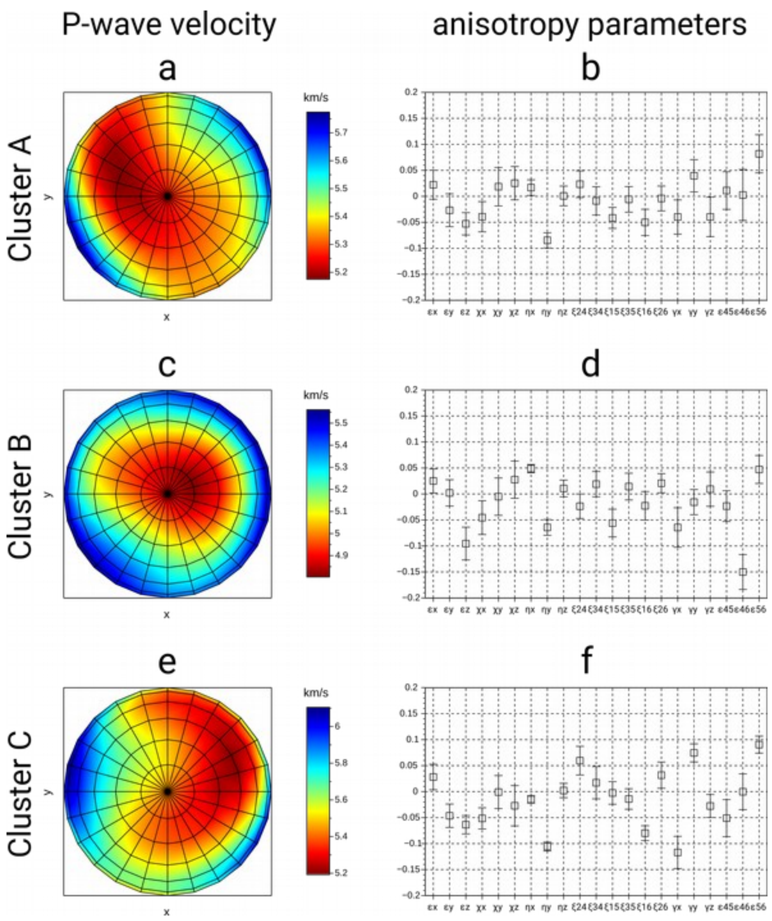


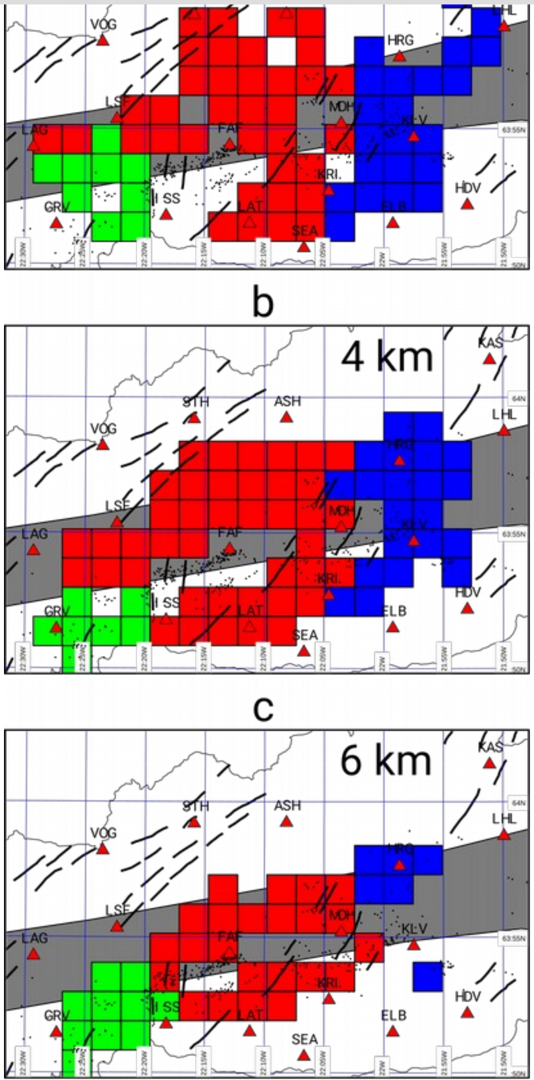
… error of anisotropy parameters is ~3 times below their nominal value.

Clustering boxes according the similarity of anisotropy



Clustering boxes according the similarity of anisotropy

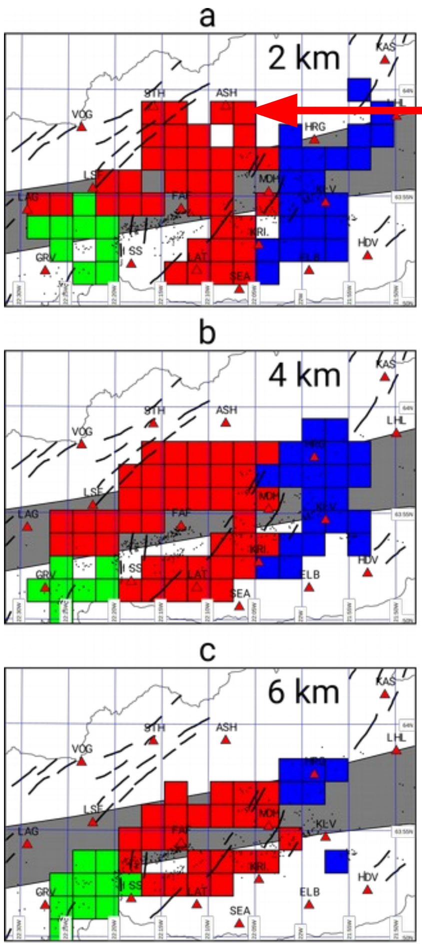


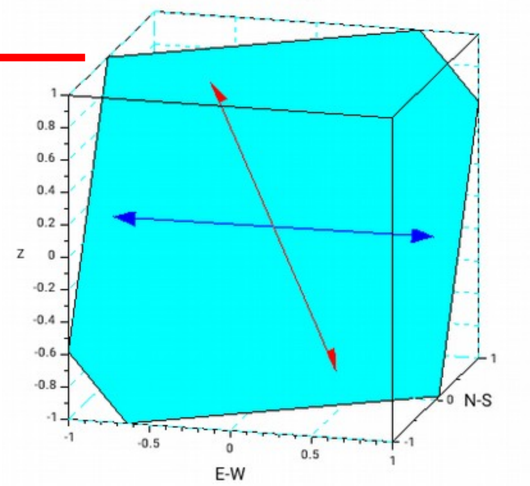


Cluster A = red
Cluster B = blue
Cluster C = green

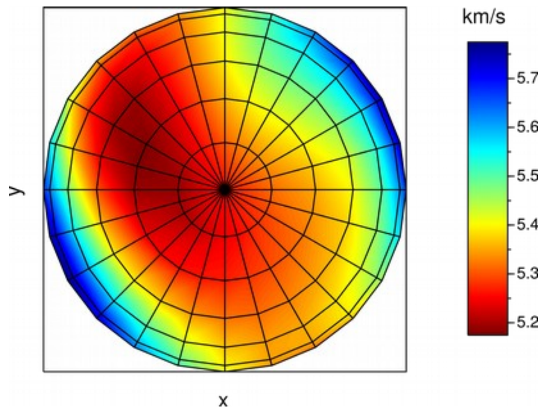
Note: Clusters are nearly independent on depth.

Possible interpretation (???)

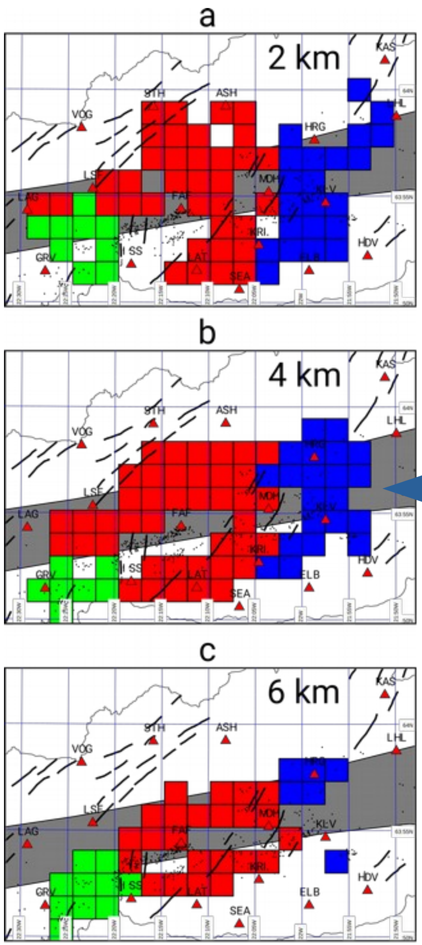




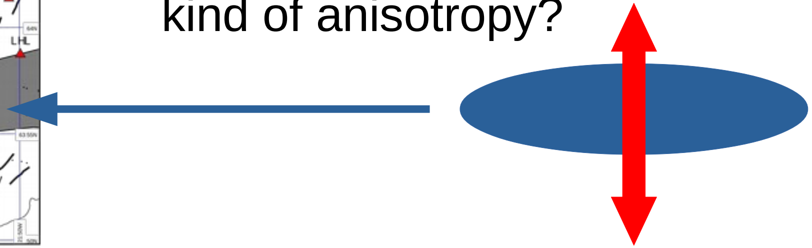
Cluster A (red)
Fast and slow P-wave directions form a subvertical plane whose strike is nearly identical to the strike of the rift (grey body in the map). The dip is oriented to SSE. Should this anisotropy reflect the existence of the rift?

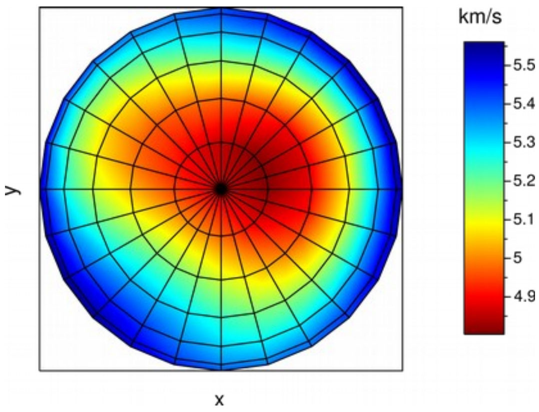


Possible interpretation (???)

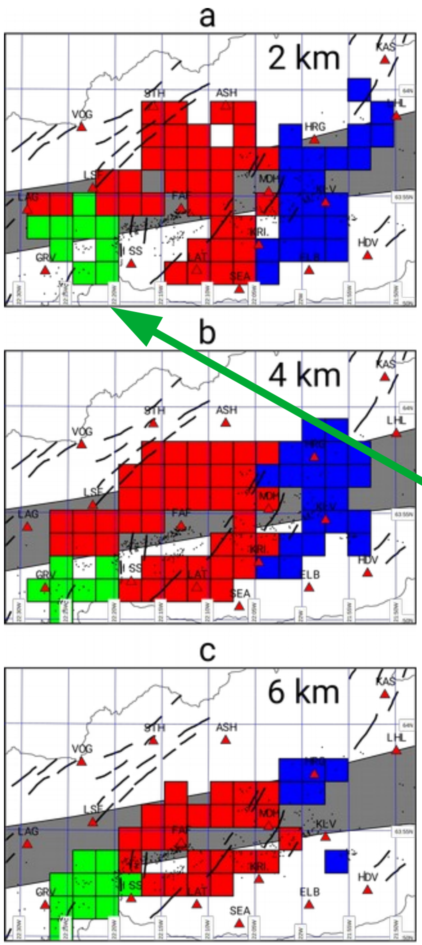


Cluster B (blue)
Anisotropy of this cluster resembles hexagonal symmetry with slow P-wave velocity axis nearly vertical. Cluster B occupies area with extraordinary water regime (e.g. Kleifarvatn Lake with oscillating water level with no surface inflow/outflow). Could subvertical water paths cause such a kind of anisotropy?

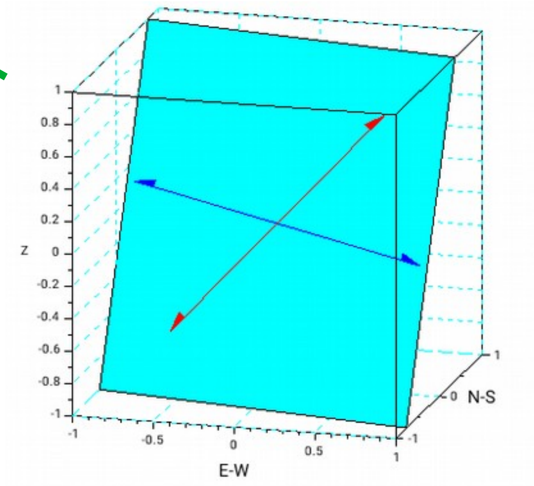


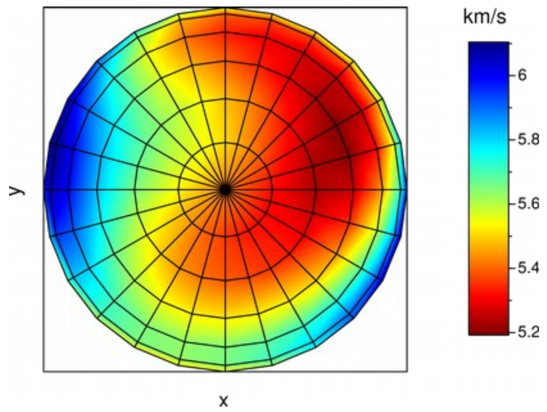


Possible interpretation (???)



Cluster C (green)
Plane constructed from slow and fast P-wave directions is dipping steeply to SSW, with the strike nearly perpendicular to the strike of majority of faults (short black-line segments in the map). Should the anisotropy in this area be a manifestation of aligned faults/dikes?





Thanks for your interest