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Cosmological Redshift and Cosmic Time Dilation in the FLRW Metric

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The paper shows that the commonly used Friedmann-Lemaitre-Robertson-Walker (FLRW) metric describing the expanding Universe must be modified to properly predict the cosmological redshift. It is proved that the change in the frequency of redshifted photons is always connected with time dilation, similarly as for the gravitational redshift. Therefore, the cosmic time runs differently at high redshifts than at present. Consequently, the cosmological time must be identified with the conformal time and the standard FLRW metric must be substituted by its conformal version. The correctness of the proposed conformal metric is convincingly confirmed by Type Ia supernovae (SNe Ia) observations. The standard FLRW metric produces essential discrepancy with the SNe Ia observations called the 'supernova dimming', and dark energy has to be introduced to comply theoretical predictions with data. By contrast, the conformal FLRW metric fits data well with no need to introduce any new free parameter. Hence, the discovery of the supernova dimming actually revealed a failure of the FLRW metric and introducing dark energy was just an unsuccessful attempt to cope with the problem within this false metric. Obviously, adopting the conformal FLRW metric for describing the evolution of the Universe has many fundamental cosmological consequences.

Keywords: expansion of the Universe, dark energy, cosmic time dilation, Type Ia supernovae, early Universe

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1 INTRODUCTION

Friedmann [1] applied the Einstein equations of General Relativity (GR) for describing the Universe and firstly showed that the space filled by uniformly distributed matter might evolve in time. The possibility that the Universe is really dynamic but not static was later supported by Lemaitre [2] and Hubble [3], who observed a systematic redshift of nearby galaxies, which was roughly proportional to their distance. This observation (called the Hubble-Lemaitre law) was interpreted as the Doppler effect produced by galaxies moving away from the Earth due to the Universe expansion.

However, the intuitive idea of the redshift as the Doppler effect was later abandoned. At present, the Universe is described by the so-called Friedmann-Lemaitre-Robertson-Walker (FLRW) metric [4–8], which introduces the scale factor $a(t)$ for describing the space expansion. The redshift is not related to the speed of the expansion as for the Doppler effect but to the ratio between sizes of the space, in which the photons were emitted and received [9, 10].

$$1 + z = \frac{a(r)}{a(e)} \tag{1}$$

where z is the redshift, and $a(e)$ and $a(r)$ are the scale factors for the emitter and receiver, respectively. Hence, the redshift of distant galaxies would be observed even in the case, when the Universe is not expanding anymore at the present epoch.

In contrast to the space coordinates, the time coordinate is assumed to be invariable during the Universe history. This is somewhat strange and surprising, because other solutions in GR such as the well-known Schwarzschild solution [11–13] involve distortions in space and time together. Therefore, some authors pointed out to other alternative theories admissible in GR and introduced more general metrics for describing isotropic homogeneous Universe evolving in time [14–16]. In this case, another function is considered in the metric tensor $g_{\alpha\beta}$, which describes the evolution of the time component g_{00}.

Among many possibilities how to define this function, the simplest way is to assume that the time and scale factors are defined by the same function $a(t)$. This option has a clear advantage, because the cosmological redshift will be defined by the same formula as the gravitational redshift

$$1 + z = \sqrt{\frac{g_{00}(r)}{g_{00}(e)}} \tag{2}$$

where $g_{00}(e)$ and $g_{00}(r)$ are the time components of the metric tensor $g_{\alpha\beta}$ for the emitter and receiver, respectively.

Introducing the same scale factor for time and space coordinates has also other advantages. Firstly, this metric evolves in time according to the so-called conformal transformation, properties of which are intensively studied in GR in recent years [17–19]. The new time coordinate is called the conformal time and the metric utilizing this time is called the conformal metric [14–16]. This metric is particularly interesting, because it leaves the Maxwell's equations unchanged from their form in the Minkowski spacetime [20–22]. The conformal metrics have also other exceptional properties and open space for new cosmological models as the Conformally Flat Space-Time Cosmology [14, 15, 23], Conformal Gravity [17, 24] or the Conformal Cyclic Cosmology [19, 25–27].

Nevertheless, introducing the conformal time into the FLRW metric is commonly viewed as a mathematical concept different from the physical cosmic time [16]. Otherwise, we have to admit a variable coordinate speed of light dependent on the scale factor $a(t)$. Although, theories of variable speed of light (VSL) exist [28, 29], they are not paid much attention, because they are against a deeply rooted concept of the speed of light as a nature constant. Nevertheless, Dicke [30] argues in his pioneering work on gravity that VSL is physically admissible. Also Dirac [31] states that "The laws may be changing, and in particular quantities which are considered to be constants of nature may be varying with cosmological time."

In this paper, the problem of cosmic time dilation and cosmological redshift in the standard FLRW metric is revisited. It is shown that time dilation and redshift observations are, actually, inconsistent with the original FLRW metric. Instead, the conformal FLRW metric should be used for describing the Universe evolution, because it predicts time dilation and redshift correctly. Cosmological consequences of this correction are discussed.

2 THEORY

2.1 FLRW Metric

The space filled by a homogenous and isotropic matter is described by the following general metric [12, 16, 22, 32]:

$$ds^2 = -A^2(t)c^2dt^2 + B^2(t)d\Sigma^2, \tag{3}$$

where $ds = cd\tau$ is the spacetime element, c is the speed of light, τ is the proper time, t is the coordinate time, Σ is the 3-dimensional coordinate in space of uniform curvature, and $A(t)$ and $B(t)$ are arbitrary functions describing time evolution of time dilation and space expansion, respectively.

The standard FLRW metric is based on the assumption of the space expansion described by the scale factor $a(t) = B(t)$ and with no time dilation $A(t) = 1$. Hence, the metric reads in the spherical coordinate system as [9, 10, 33].

$$\begin{gathered} ds^2 = -c^2dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \\ d\Omega^2 = d\Theta^2 + \sin^2\Theta d\phi^2, \end{gathered} \tag{4}$$

k is the curvature index of the space, r is the comoving distance, and Θ and ϕ are the spherical angles.

An alternative to **Eq. 4** is the so-called conformal form of the FLRW metric [16], which assumes the same factor $a(t)$ for time dilation and space expansion, $A(t) = B(t) = a(t)$,

$$ds^2 = a^2(t)\left(-c^2\, dt^2 + \frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{5}$$

where time t has a different physical meaning than in **Eq. 4** being often denoted as η.

Obviously, Einstein's equations do not constrain functions $A(t)$ and $B(t)$ in **Eq. 3** and they do not give us any preference between **Eq. 4** for the standard FLRW metric and **Eq. 5** for the conformal FLRW metric. Both metrics are based on the assumption of perfect isotropy and homogeneity and they satisfy the GR equations.

2.2 Coordinate Freedom of Choosing Time

We can see that **Eq. 5** is obtained from **Eq. 4** by a simple transformation

$$dt = a(t)d\eta, \tag{6}$$

where η is called the conformal (comoving) time and t is the proper time. Commonly, the conformal time η is considered as a mathematical concept different from the physical coordinate time. In this case, **Eqs 4**, **5** are physically equivalent, because we applied just rescaling of time using **Eq. 6** and the Einstein equations are coordinate invariant [12, 34].

However, we should be aware that the coordinate invariance of the Einstein equations does not mean that we can rescale time and space coordinates arbitrarily with no physical consequences. The physically meaningful coordinates should be identified with the "cosmological coordinate system," in which all fundamental bodies are in rest [14, 15, 20, 21]. Also, we cannot mix comoving and proper coordinates in the metric. If we ignore this condition and do not distinguish between comoving and proper coordinates, **Eqs 4**, **5** can possibly describe the static Universe, provided distance r is substituted by the proper distance R as

$$dr = \frac{dR}{a(t)}. \tag{7}$$

Hence, the key for understanding **Eqs 4**, **5** is to define, which quantities are physical (being related to the cosmological coordinate system) and which quantities describe just an arbitrary coordinate with no physical meaning. If r is the comoving distance, **Eqs 4**, **5** do not describe the static Universe but the expanding Universe.

Similarly, if the conformal time η is the comoving time measured by clocks in the cosmological coordinate system, then **Eqs 4**, **5** define two physically different Universe models. This is obvious, because **Eq. 4** assumes the cosmic time being invariant of the space expansion, while **Eq. 5** assumes the cosmic time being dependent on the space expansion. Consequently, the coordinate speed of light is invariant in **Eq. 4** but it depends on $a(t)$ in **Eq. 5**, see **Appendix A**. Since both equations are admissible in GR, the correct form of the metric of the cosmological coordinate system must be found from observations. Primarily, the correct metric should satisfactorily explain observations of the cosmological redshift.

2.3 Cosmological Redshift Inconsistency

The cosmological redshift in the standard FLRW metric is commonly explained as the change of the photon wavelength due to the space expansion [9, 10, 33, 35, 36]. The common derivation in textbooks is as follows. Light travels along the null geodesic, $ds = cd\tau = 0$, hence

$$c^2 dt^2 = a^2(t) dl^2, \tag{8}$$

where dl is the element of the comoving distance. Consequently,

$$\frac{cdt}{a(t)} = dl. \tag{9}$$

Suppose the distant galaxy emits photons at constant rate Δt_e and with wavelength λ_e. The photons are observed at rate Δt_r and with wavelength λ_r. The first photon is emitted at time t_e and received at time t_r. Taking into account that the comoving distance between the galaxy and the observer is the same for the two successive photons

$$\int_{t_e}^{t_r} \frac{cdt}{a(t)} = \int_{t_e+\Delta t_e}^{t_r+\Delta t_r} \frac{cdt}{a(t)} \tag{10}$$

and subtracting the integral

$$\int_{t_e+\Delta t_e}^{t_r} \frac{cdt}{a(t)} \tag{11}$$

we get

$$\int_{t_e}^{t_e+\Delta t_e} \frac{cdt}{a(t)} = \int_{t_r}^{t_r+\Delta t_r} \frac{cdt}{a(t)} \tag{12}$$

Since the scale factor $a(t)$ varies slowly and does not change much during emission and observation of the two successive photons, we write

$$\frac{1}{a(t_e)} \int_{t_e}^{t_e+\Delta t_e} cdt = \frac{1}{a(t_r)} \int_{t_r}^{t_r+\Delta t_r} cdt. \tag{13}$$

Hence,

$$\frac{d_e}{a(t_e)} = \frac{d_r}{a(t_r)} \tag{14}$$

where $d_e = c\Delta t_e$ and $d_r = c\Delta t_r$ are the distances between two successive photons at the emitter and the receiver, respectively. Subsequently, we can conclude that the wavelengths of photons λ_e and λ_r obey the same relation

$$\frac{\lambda_e}{a(t_e)} = \frac{\lambda_r}{a(t_r)} \tag{15}$$

This derivation is not, however, correct. Using **Eq. 13**, we can also obtain the following equation

$$\frac{\Delta t_e}{a(t_e)} = \frac{\Delta t_r}{a(t_r)} \tag{16}$$

which indicates that the coordinate time depends on the scale factor $a(t)$. Obviously, **Eq. 16** is inconsistent with the standard FLRW metric described by **Eq. 4**, where the coordinate time is invariant. Alternatively, we can keep the coordinate time independent of the scale factor, but then we have to assume that the light speed c depends on the scale factor $a(t)$ and we have to distinguish between the light speed in the emitter, c_e, and in the receiver, c_r. This is again inconsistent with **Eq. 4**.

The basic difficulty with the above derivation of redshift-dependent wavelengths of photons lies in an incorrect definition of the wavelength as distance between two different spacetime events, see **Appendices B**, **C**. Obviously, the distance must be measured at one coordinate system, but not as a distance between points in two different coordinate systems connected with two photons measured at different times. Once we consider two photons travelling along the same ray path with distance d between them at the same coordinate time, the effect of increasing the distance between photons during the space expansion disappears. After any time t, both photons travel the same distance along the same ray, and consequently the distance between them keeps time independent, see **Appendix B**.

Mathematically, we modify **Eq. 10**, in which we do not assume the equality of the comoving distance but the equality of the light travel distance of the photons propagating along the same raypath from the emitter to the receiver:

$$\int_{t_e}^{t_r} cdt = \int_{t_e+\Delta t}^{t_r+\Delta t} cdt. \tag{17}$$

Using the same logic as above, we obtain that if time and speed of light is not changing, the wavelength of photons does not change. Hence, two successive photons travelling along the same raypath keep their mutual proper distance constant and independent of redshift. However, the proper distance between two photons travelling along two parallel rays at the same time depends on redshift and increases with the space expansion. This is because the comoving distance between two photons moving along parallel raypaths is constant, hence the proper distance must increase with the space expansion, see **Appendix C**. Only the proper distance between two successive photons travelling along the same ray does not change, see **Appendix B**.

The above derivation proves that the standard FLRW metric cannot be applied to the Universe, because it does not predict the cosmological redshift. The cosmological redshift can be observed only if the cosmic time depends on the scale factor $a(t)$ and it runs differently at high redshift than at present. Therefore, the cosmological redshift is not a consequence of the space expansion but of time dilation. A disputable character of the original FLRW metric is also indicated by comparing this metric with other solutions in GR, where the expansion/contraction of space is tightly connected with time dilation. If we insist on no time dilation, no redshift will be observed.

The variability of the cosmic time during the Universe evolution would be supported by the fact that the mass density in the Universe is time dependent. At previous epochs, the Universe was much denser and the gravitational field much stronger. Going back in time to high redshifts is analogous to the case, when an observer moves towards the black hole. According to the Schwarzschild solution, the coordinate time for the observer close to the black hole runs differently than for the observer far from the black hole. Similarly, the coordinate time must run differently at the high redshift Universe than at the present epoch. Consequently, assuming that the Universe expands but the cosmic time is invariant is physically unjustified.

Hence, the correct metric is the conformal form of the FLRW metric described by **Eq. 5** and the cosmological redshift obeys the same formula as the gravitational redshift:

$$\frac{\nu_e}{\nu_r} = 1 + z = \sqrt{\frac{g_{00}(r)}{g_{00}(e)}} \tag{18}$$

where z is the redshift, ν_e and ν_r are the frequencies of the photon at the emitter and receiver, and $g_{00}(e)$ and $g_{00}(r)$ are the time components of the metric tensor $g_{\alpha\beta}$ at the emitter and receiver, respectively.

2.4 Properties of the Conformal FLRW Metric

The conformal FLRW metric is essentially different from the original FLRW metric with fundamental physical consequences:

- **Eq. 5** implies that the comoving speed of light is constant but the proper speed of light depends on redshift. Hence, the volume of the Universe and distance between galaxies were smaller at high redshift, but photons emitted by a galaxy reach a neighbouring galaxy after the same time at high redshift as well as at the present epoch. In other words, this Universe model is conformal with the static Universe.
- The frequency ν_e of photons emitted at redshift z is higher than the frequency ν_r of photons received as:

$$\frac{\nu_e}{\nu_r} = 1 + z. \tag{19}$$

- Not only the frequency ν of photons but also the rate of photons increases with redshift as $(1 + z)$.
- The proper speed of light c in the cosmological coordinate system decreases with redshift as $(1 + z)^{-1}$.
- The wavelength λ_e of photons emitted at redshift z is shorter than the wavelength λ_r of photons received as:

$$\frac{\lambda_e}{\lambda_r} = (1 + z)^{-2}. \tag{20}$$

This includes a decrease of frequency ν and an increase of the speed of light c with cosmic time.

2.5 Friedmann Equations Revisited

If the expansion of the Universe is described by the conformal FLRW metric, the Friedmann equations must be modified. The standard Friedmann equations for the pressureless fluid read [10, 33].

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{1}{3}\Lambda c^2, \tag{21}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{1}{3}\Lambda c^2, \tag{22}$$

where $a = (1 + z)^{-1}$ is the scale factor, G is the gravitational constant, ρ is the mean mass density, k/a^2 is the spatial curvature of the Universe, and Λ is the cosmological constant.

In order to express the Friedmann equations for the conformal FLRW metric, we have to substitute time t by the conformal time η and time derivative $\dot{a} = da/dt$ by $a' = da/d\eta = a\dot{a}$. Hence, the conformal Friedmann equations read

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3}\rho a^2 - kc^2, \tag{23}$$

$$\frac{a''}{a} = -\frac{4\pi G}{3}\rho a^2, \tag{24}$$

where we omitted the cosmological constant, because it was inserted into **Eqs 21** and **22** artificially in order to fit Type Ia supernova observations. Considering the matter-dominated Universe, we get

$$\frac{8\pi G}{3}\rho = H_0^2\,\Omega_m a^{-3} \tag{25}$$

and **Eq. 23** reads

$$H^2(a) = H_0^2\left(\Omega_m a^{-1} + \Omega_k\right) \tag{26}$$

with the condition

$$\Omega_m + \Omega_k = 1, \tag{27}$$

where $H(a) = a'/a$ is the Hubble parameter, H_0 is the Hubble constant, Ω_m is the normalized matter density, and Ω_k is the normalized space curvature. Since this model is basically the Einstein-de Sitter (EdS) model but applied to the conformal FLRW metric, it will be called as the "conformal EdS model" in contrast to the standard EdS model based on the original FLRW metric.

3 SUPERNOVAE OBSERVATIONS

The correctness of **Eq. 26** for the time evolution of the Universe can be checked by Type Ia supernova (SNe Ia) observations, which provide the most accurate measurements of cosmological distances and of the expansion history of the Universe. A discrepancy between the supernova observations and the predictions of the standard EdS model was called the "supernovae dimming" [37, 38], and led to reintroducing the cosmological constant Λ into the Einstein and Friedmann equations. The observation of the unexpected SNe Ia dimming motivated large-scale systematic searches for SNe Ia and resulted in a rapid extension of supernovae compilations.

The current supernovae compilations Union2.1 [39–44], and Pantheon [45, 46] comprise of hundreds of SNe Ia discovered and spectroscopically confirmed. The Pantheon dataset is the most accurate SNe Ia compilation at present. Every SN Ia is described by its apparent rest-frame B-band magnitude m_B, the absolute B-band magnitude M_B, the stretch parameter x_1, and the colour parameter c. These parameters are used in the Tripp formula [47, 48] for calculating the redshift-dependent distance modulus $\mu(z)$, which serves for testing the cosmological models,

$$\mu = m_B - M_B + \alpha x_1 - \beta c \tag{28}$$

where coefficients α and β are the global nuisance parameters to be determined when seeking an optimum cosmological model. The expansion history is calculated from μ using the following equations,

$$\mu = 25 + 5\log_{10}(d_L)\,,\ d_L = (1+z)\int_0^z \frac{c dz'}{H(z')} \tag{29}$$

where d_L is the luminosity distance expressed for the flat Universe. The Hubble function $H(z)$ is expressed for the flat Universe described by the standard ΛCDM model as

$$H^2(z) = H_0^2\left[\Omega_m (1+z)^3 + \Omega_\Lambda\right], \tag{30}$$

by the standard EdS model as

$$H^2(z) = H_0^2\left[\Omega_m (1+z)^3 + \Omega_k (1+z)^2\right], \tag{31}$$

and by the conformal EdS model as

$$H^2(z) = H_0^2\left[\Omega_m (1+z) + \Omega_k\right]. \tag{32}$$

While the ΛCDM model contains dark energy Ω_Λ as a free parameter, which must be adjusted by fitting with the SNe Ia observations, the conformal EdS model requires no free parameter for the flat Universe, and the curvature parameter Ω_k is needed for a curved Universe. Since the Universe is nearly flat, this parameter should be close to zero and can be determined from other independent observations. Model-independent methods for estimating Ω_k are based on reconstructing the comoving distances by Hubble parameter data and comparing with the luminosity distances [49–51], on the angular diameter distances [52], on strongly gravitational lensed SNe Ia [53] or on gravitational waves [54]. The authors report the curvature term Ω_k ranging between −0.3 and −0.1 indicating that the Universe is nearly flat and closed.

Figure 1 shows a comparison of the SNe Ia measurements with predictions of the ΛCDM model and the standard and conformal EdS models. The standard EdS model is in a visible disagreement with the SNe Ia measurements and this disagreement led to developing the ΛCDM model by introducing the normalized density of dark energy Ω_Λ into **Eq. 30** to get a satisfactory fit. Strikingly, the conformal EdS model defined by **Eq. 32** fits data equally well as the ΛCDM model with no assumption on dark energy (see **Figure 2**). This confirms that the solution of the puzzle with the supernovae dimming does not lie in introducing dark energy but in correcting the metric used in the Friedmann equations.

4 DISCUSSION

The Friedmann equations introduce the expansion of the Universe and form fundamentals of modern cosmology. Intuitively, the space expansion can explain the cosmological redshift, because the distant galaxies are moving away due to the expansion and we observe their light distorted by the Doppler effect. This was probably the motivation for describing the Universe by the standard FLRW metric. The problem is, however, more involved, and we know that the cosmological redshift is not due to the Doppler effect but due to distortion of the spacetime described by GR. The redshift of distant galaxies would be observed even for a non-expanding Universe at the present epoch. From this point of view, there is no clear argument, why the standard FLRW metric introduces just the space expansion with no time dilation.

In fact, it is surprising to assume distortion of space only, because other solutions in GR such as the well-known Schwarzschild solution involve distortions in space and time together. At previous epochs, the Universe was much denser and the gravitational field much stronger, hence going back in time to high redshifts is analogous to an observer moving towards the black hole. Since the coordinate time runs differently close to and far from the black hole, we can expect to observe a similar effect when comparing clocks at the high redshift Universe and at the present epoch.

In addition, the assumption of no time dilation during the Universe evolution is not strange only from the theoretical point of view but it is also in contradiction with astronomical observations. The existence of cosmic time dilation and its real physical nature is supported by observations of gamma ray-bursts [56–58] and Type Ia supernovae light curves [59, 60]. For example, Zhang [61] studied a sample of 139 SWIFT long gamma-ray bursts (GRBs) with redshift $z \leq 8.2$ and obtained a significant correlation between their duration and redshift. Similarly, Littlejohns and Butler [62] analysed 232 GRBs detected by the Swift/Burst Alert Telescope (BAT) and revealed that the observed durations are consistent with cosmic time dilation. As regards supernovae, the SNe Ia display rather uniform light curves and thus they can be used as local clocks. The spectral evolution of the light curves and stretching of time in the observer frame was disclosed by many authors [59, 63–65], and corrections for time dilation are now routinely applied to the SNe Ia data [60, 66].

The re-examination of light propagation in space defined by the standard FLRW metric reveals another severe contradiction with observations: this metric actually does not predict the cosmological redshift. This is surprising and against the common opinion that the standard FLRW metric produces the cosmological redshift. However, it is shown that the mathematical derivation originally proposed by Lemaitre [2] and repeated in textbooks is not correct. Lemaitre [2] analysed the change of the wavelength of photons

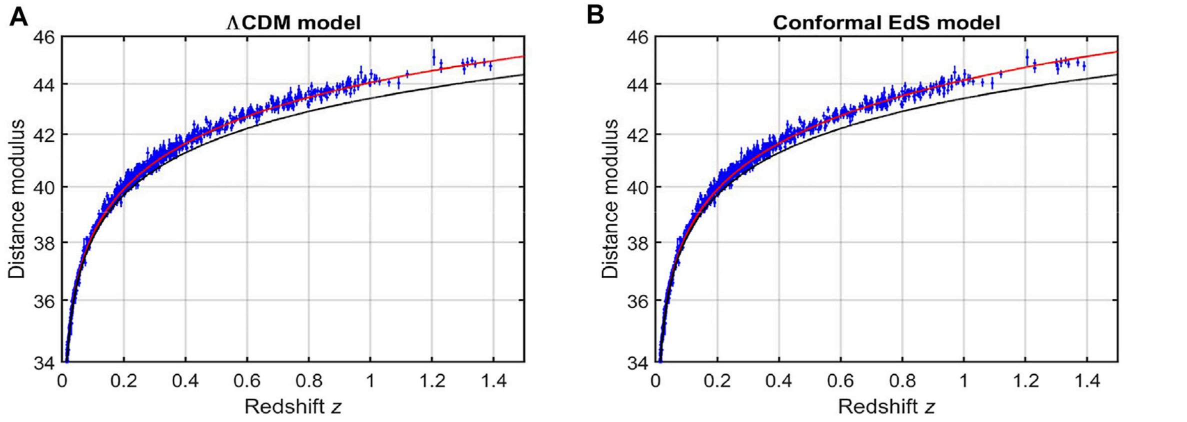

FIGURE 1 | The Hubble diagram with Type Ia supernovae observations. Blue dots show measurements of the SNe Pantheon compilation [45, 46]. The red line in **(A)** shows the ΛCDM model described by **Eq. 30** with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. The red line in **(B)** shows the conformal EdS model described by **Eq. 32** with $\Omega_m = 1.2$ and $\Omega_k = -0.2$. The black line in **(A,B)** shows the standard EdS model described by **Eq. 31** with $\Omega_m = 1.0$ and $\Omega_k = 0$. The Hubble constant is $H_0 = 69.8$ km s^{-1} Mpc^{-1}, obtained from observations of the SNe Ia data with a red giant calibration [55].

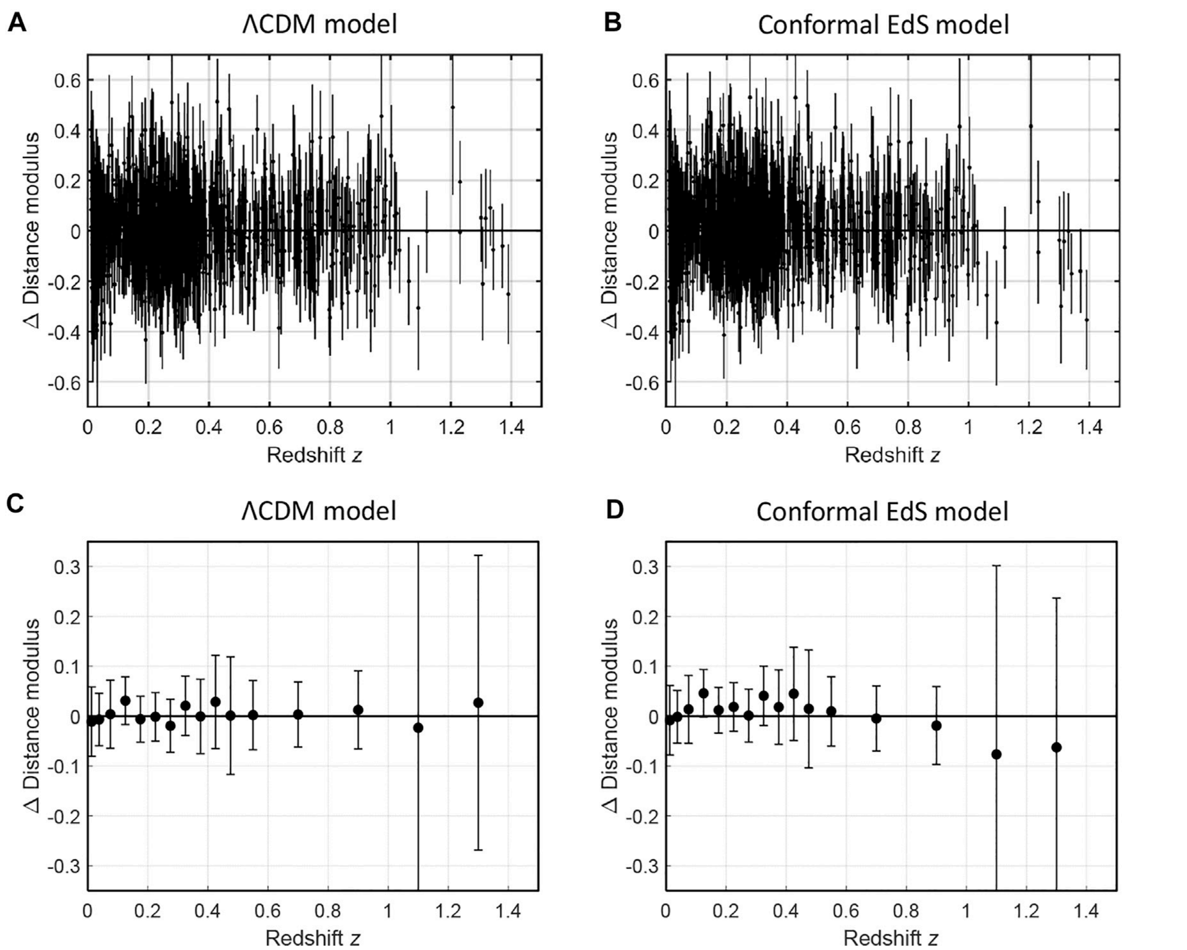

FIGURE 2 | Residual Hubble plots for **(A,B)** the individual SNe Ia data and **(C,D)** the binned SNe Ia data. **(A,C)** The flat ΛCDM model, **(B,D)** the conformal EdS model. For parameters of the models, see caption of **Figure 1**. The error bars in **(C,D)** show the 99% confidence intervals. Data are taken from the SNe Pantheon compilation [45, 46].

propagating in expanding space and he came to a wrong conclusion that the wavelength of photons must increase, similarly as the proper distance between objects in rest. An increasing wavelength of photons is then transformed into the change of their frequency under the assumption of the constant speed of light. Since this derivation gave intuitively acceptable results, there was no reason to critically check its correctness by other cosmologists.

A correct analysis shows, however, that the wavelength of photons does not increase and the frequency of photons is constant during the space expansion defined by the standard FLRW metric. The change in the frequency of photons is always connected with time dilation and with a variation of the time metric g_{00} in GR, similarly as for the gravitational redshift. Therefore, the standard FLRW metric must be substituted by the conformal FLRW metric that predicts the cosmic time dilation and the cosmological redshift properly. Consequently, the cosmic time should be identified with the conformal time and the space-time evolution of the Universe should be described by the conformal FLRW metric only.

Obviously, we can ask a question: why atoms radiate photons with the same (rest-frame) frequency at all redshifts and why this frequency is not affected by time dilation? The answer is straightforward: the frequency of emitted photons is independent of redshift, because it depends on quantized energy levels of electrons in atoms and these energy levels are redshift independent. Once the photon is emitted, its frequency decreases due to time dilation when photon propagates along the ray path from the emitter to the receiver. Since the comoving speed of light is constant, the proper speed of light must be variable. In this way, the emitted photons with frequency ν have shorter proper wavelengths at high redshift than the photons with the same frequency ν but emitted at the present epoch.

The correctness of the conformal FLRW metric is convincingly confirmed by SNe Ia observations. In fact, observations of the SNe Ia were originally proposed by Riess et al. [37] and Perlmutter et al. [38] for testifying the existing cosmological model and the SNe Ia observations surprisingly revealed essential discrepancy between theoretical predictions and measurements. However, instead of questioning the validity of the standard FLRW metric and the Friedmann equations, Riess et al. [37] and Perlmutter et al. [38] introduced a free parameter into the Friedmann equations to comply them with data. In this way, the model is capable to fit the SNe Ia observations, but at the cost of introducing a physically controversial concept of dark energy. By contrast, the EdS model based on the conformal FLRW metric fits the SNe Ia data with no need to introduce any new free parameter.

An argument that dark energy is not physical, but originates in the applied standard FLRW metric is used also by other authors [67–70]. For example, the accelerated expansion could be an artefact of neglecting inhomogeneity of the Universe [71–75] as proposed in the Swiss-cheese cosmology [76–78] or in the timescape cosmology [79–81]. The SNe Ia dimming can partly be a result of cosmic opacity neglected in interpretations of the SNe Ia luminosity [82–85]. By contrast, here I show that the essential difficulty with the standard FLRW metric is not in the oversimplification of the model by assuming perfect homogeneity and isotropy of the Universe, but in false neglecting time dilation during the Universe history. The results indicate that anisotropy, heterogeneity and opacity of the Universe produce probably only the second-order effects in observations.

5 CONCLUSION

In summary, we conclude that the conformal FLRW metric is the only correct metric for describing the evolution of the Universe, which can predict the cosmological redshift and time dilation properly. If the time rate is independent of the expansion of the Universe as in the standard FLRW metric, the frequency of photons cannot change during the expansion. Therefore, the variable rate of time during the expansion is inevitable and implies the following fundamental consequences:

(1) The gravitational and cosmological redshifts are calculated by the same formula and describe the same physical process. Both redshifts reflect a distortion of time produced by changes in the gravitational field. While the gravitational redshift originates in spatial variations of the gravitational field, the cosmological redshift originates in temporal variations of the gravitational field.
(2) The metric describing the evolution of the Universe is conformal with the static model. This metric leaves the Maxwell's equations unchanged from their form in the Minkowski spacetime [20–22].
(3) The conformal FLRW metric predicts correctly the cosmological redshift: the frequency of photons increases with redshift as $(1 + z)$. Not only the frequency of photons but also the rate of photons increases with redshift as $(1 + z)$ due to time dilation. The real physical nature of cosmic time dilation is supported by observations of gamma ray-bursts [56–58] and Type Ia supernovae light curves [59, 60, 66].
(4) The comoving speed of light is constant. The proper speed of light decreases with redshift as $(1 + z)^{-1}$. Hence, the speed of light is not a nature constant but it varies being dependent on the scale factor $a(t)$ [28, 86]. Consequently, distance between galaxies changes with redshift, but photons emitted by a galaxy reach a neighbouring galaxy after the same time at high redshift as well as at the present epoch. The wavelength of photons does not decrease with redshift as $(1 + z)^{-1}$ as assumed in the standard FLRW metric but it decreases with redshift as $(1 + z)^{-2}$.
(5) The conformal FLRW metric fits the SN Ia observations with no need to introduce dark energy into the Einstein and Friedmann equations. The dark energy is an artefact of the erroneous metric used for describing the evolution of the Universe. Consequently, no repulsive forces produced by dark energy and acting against gravity are present in the corrected Friedmann equations. Since the only force considered in the Friedmann equations is gravity, the expansion of the Universe is decelerating at the present epoch.

DATA AVAILABILITY STATEMENT

Publicly available datasets were analyzed in this study. This data can be found here: https://archive.stsci.edu/prepds/ps1cosmo/.

AUTHOR CONTRIBUTIONS

VV is the only author of all presented results.

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APPENDIX A: COORDINATE SPEED OF LIGHT IN THE STANDARD AND CONFORMAL FLRW METRICS

Let us assume light propagating in the space described by the standard FLRW metric, see **Eq. 4**. The equation of the null geodesics for photons, $ds^2 = 0$, yields

$$cdt = a(t)dl, \tag{A1}$$

where dl is the element of the comoving distance. The comoving speed of light v reads

$$v = \frac{dl}{dt} = \frac{c}{a(t)}, \tag{A2}$$

and the proper speed of light $\tilde{v}$ is

$$\tilde{v} = \sqrt{v^i v_i} = \sqrt{v^i v^i g_{ii}} = a(t)v = c. \tag{A3}$$

If light propagates in the space described by the conformal FLRW metric described by **Eq. 5**, the equation of the null geodesics for photons, $ds^2 = 0$, yields

$$cdt = dl. \tag{A4}$$

Hence, the comoving speed of light v is

$$v = \frac{dl}{dt} = c, \tag{A5}$$

and the proper speed of light $\tilde{v}$ is

$$\tilde{v} = \sqrt{v^i v_i} = \sqrt{v^i v^i g_{ii}} = a(t)v = a(t)c. \tag{A6}$$

The dependence of $\tilde{v}$ on the scale factor $a(t)$ in **Eq. A6** is a trivial consequence of **Eq. A5** expressing that the speed of light is constant in the comoving coordinates. Since the proper speed of light is the actually measured speed of light, **Eqs A3**, **A6** predict essentially different behaviour of light in the standard and conformal FLRW metrics.

APPENDIX B: DISTANCE BETWEEN TWO SUCCESSIVE PHOTONS TRAVELLING ALONG THE SAME RAYPATH

Let us assume two photons propagating in the space described by the standard FLRW metric, see **Eq. 4**. We will consider the case of two successive photons travelling along the same raypath with time delay Δt between them. The photons are emitted by a common source situated at the origin of coordinates and they travel in the space along the x-axis for time T to reach a receiver. The equations of the null geodesics for the photons, $ds^2 = 0$, yield

$$cdt = a(t)dx\,,\; cdt' = a(t')dx', \tag{B1}$$

where $t' = t + \Delta t$. The initial comoving coordinates of photons at the initial time t_0 are taken as

$$x_0 = \int_{t_0}^{t_0+\Delta t} \frac{cdt}{a(t)}\,,\; y_0 = 0\,,\; z_0 = 0, \tag{B2}$$

$$x_0' = 0\,,\; y_0' = 0\,,\; z_0' = 0, \tag{B3}$$

and the comoving distance d_0 between the photons at time t_0 reads

$$d_0 = x_0 - x_0' = \int_{t_0}^{t_0+\Delta t} \frac{cdt}{a(t)} = \frac{\tilde{d}_0}{a_0} \tag{B4}$$

where $\tilde{d}_0$ is the proper distance between the photons at time t_0 defined as

$$\tilde{d}_0 = \int_{t_0}^{t_0+\Delta t} cdt = c\Delta t \tag{B5}$$

and we assumed in **Eq. B4** that the scale factor $a(t)$ does not change much during the time interval Δt. Once the second photon reaches the receiver, we get

$$d_T = x_T - x_T' = \int_{t_0+T}^{t_0+T+\Delta t} \frac{cdt}{a(t)} = \int_{T}^{T+\Delta t} \frac{cdt}{a(t)} = \frac{\tilde{d}_T}{a_T} \tag{B6}$$

where a_T is the scale factor at time $t_0 + T$ and $\tilde{d}_T$ is the proper distance between the photons at time $t_0 + T$

$$\tilde{d}_T = \int_{t_0+T}^{t_0+T+\Delta t} cdt = c\Delta t. \tag{B7}$$

Comparing **Eqs B5**, **B7**, we see that the proper distance between two successive photons is constant and independent of the scale factor $a(t)$. Consequently, the wavelength of photons cannot change with the scale factor $a(t)$ in the standard FLRW metric.

APPENDIX C: DISTANCE BETWEEN TWO PHOTONS TRAVELLING ALONG PARALLEL RAYPATHS

Let us assume two photons propagating in the space described by the standard FLRW metric, see **Eq. 4**. We will consider the case of two photons emitted at the same time by two different sources and travelling along two parallel rays. The photons travel in the space along the x-axis and need time T to reach their receivers. The equations of the null geodesics for the photons, $ds^2 = 0$, yield

$$cdt = a(t)dx\,,\; cdt = a(t)dx'. \tag{C1}$$

The initial comoving coordinates of photons at the initial time t_0 are taken as

$$x_0 = 0\,,\; y_0 = d_0\,,\; z_0 = 0, \tag{C2}$$

$$x_0' = 0\,,\; y_0' = 0\,,\; z_0' = 0. \tag{C3}$$

Hence, the initial comoving distance between the two photons is d_0. After elapsing time T, we get

$$x_T = \int_{t_0}^{t_0+\Delta t} \frac{cdt}{a(t)}\,,\; y_0 = d_0\,,\; z_0 = 0, \tag{C4}$$

$$x'_T = \int_{t_0}^{t_0+\Delta t} \frac{cdt}{a(t)}\,,\ y'_0 = 0\,,\ z'_0 = 0, \tag{C5}$$

and the comoving distance d_T between the photons at time $t_0 + T$ reads

$$d_T = d_0. \tag{C6}$$

Consequently, the proper distances $\tilde{d}_0$ and $\tilde{d}_T$ between the two photons at times t_0 and $t_0 + T$ read

$$\tilde{d}_0 = a_0 d_0\,,\ \tilde{d}_T = a_T d_T, \tag{C7}$$

implying that the proper distance between the photons linearly increases with the increasing scale factor $a(t)$.